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MATHEMATICS
FOR
ELECTRICAL STUDENTS

BY

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PREFACE

THE purpose of these two brief texts is to give to industrial workers and students who have not completed high school, that part of algebra, geometry, and trigonometry which they will need in their work. The parts of algebra, geometry, and trigonometry necessary only for students going on to college are omitted. The texts are planned to present the material in a manner that will enable the student to master it with a minimum of outside help, and it is believed, therefore, that they are suitable for use with evening school and continuation classes, or wherever the work is largely individual.

The contents of the course may briefly be outlined as follows:

1. Equations.
2. Slide Rule.
3. Formulas.
4. Positive and Negative Numbers.
5. Proportion.
6. Quadratic Equations.
7. Simultaneous Equations.
8. Graphs.
9. Commonly used Geometric Facts (without proof).
10. Logarithms.
11. Right Triangles.
12. Oblique Triangles.

The work is published in two volumes; one especially adapted to those interested in shopwork and drafting, the other to those interested in electrical work. The theoretical parts of the two are identical, but the one has applications from the shop and the other from electrical work.

The books are not shop or electrical texts and cannot be expected to instruct students along shop or electrical lines. They are mathematics books, written for shop or electrical students and workers, with applications drawn from the shop and electrical work. The main purpose of each is to give not merely a knowledge of mathematics, but to furnish that understanding of mathematics which will increase the student's practical working knowledge.

The chapter on geometry is intended to state definitely the geometrical facts that most students know in a more or less general way. All of the geometry ordinarily needed for trigonometry and its uses is given, and the less evident theorems are illustrated by figures. The student should study the entire chapter and solve the problems at the end as a test of his understanding of geometrical facts and processes.

The chapter on the slide rule is not a discussion of the theory of the rule but specific directions for its use.

The method of presentation is as direct and definite as possible, so that the student can study the text by himself, and master it with little outside help. In courses where instruction on the slide rule is given, the student should be urged to use the rule as much as possible, especially in the solution of formulas and in checking logarithmic problems.

Many of the self-evident and little used definitions are omitted from the text for the sake of clearness. A dictionary of terms used in elementary mathematics is given in the

appendix. The attention of students should be called to this dictionary, and they should be urged to use it.

Tables of formulas, square roots, and decimal equivalents are given in the appendix for the convenience of the student.

Should any students, after completing this text, desire to fulfill the college entrance requirements in mathematics, they will find that the knowledge gained from this course will enable them to cover the work in a regular college entrance course rapidly and efficiently, or the following topics added to the work of this course will meet the requirements:

Removal of Parentheses.
Multiplication (more advanced problems).
Division (Polynomial by Polynomial).
Factoring.
Fractional Equations (Polynomial Denominator).
Literal Equations.
Radicals.
Equations Containing Radicals.
Fractional, Zero, and Negative Exponents.
Formal Geometry.
Relation between Trigonometric Functions.
Functions of the Sum and Difference of two Angles.
Functions of twice an Angle and of half an Angle.
Proof of the Law of Sines, Law of Cosines, Law of Tangents.

H. M. K.
C. J. L.

DETROIT, 1921.

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MATHEMATICS FOR ELECTRICAL STUDENTS

CHAPTER I

THE EQUATION

1. Definition. Scales will balance only when equal weights are placed in the pans of the scales. The fact that the 10-lb. weight in one pan, Fig. 1, is exactly balanced by the 5-, 3-, and 2-lb. weights in the other pan may be expressed,

$$5+3+2=10.$$

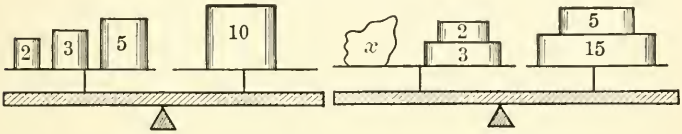

Fig. 1. Fig. 2.

In the same manner, the balance of the scales when the unknown weight (x) and 5 lbs. are placed in one pan and 20 lbs. in the other, Fig. 2, may be expressed by the equation,

$$x+5=20.$$

The weight of (x) may be found by removing 5 lbs. from each pan of the scales, leaving (x) in one pan and

15 lbs. in the other and *the scales still in balance.* This balance may be expressed by the equation,

$$x = 15.$$

The equation $x+5=20$ is a statement that two expressions are equal. The expressions $x+5$ and 20 are called **members** of the equation.

To solve an equation is to find the value of the unknown which is expressed in the equation by a letter.

2. Subtracting from Both Members of an Equation.

Example. Solve $x+5=20$

$$x+5=20$$
$$x=15 \quad \text{(subtracting 5 from both members of the equation)}$$

EXERCISE 1

Solve for the unknown in each equation:

1. $x+7=12$.	Ans. 5.	6. $x+3.3=7.8$.	Ans. 4.5.
2. $x+9=17$.	8.	7. $a+2\frac{1}{2}=3\frac{3}{4}$.	$1\frac{1}{4}$.
3. $y+2=3\frac{1}{2}$.	$1\frac{1}{2}$.	8. $x+31\frac{3}{8}=42\frac{5}{8}$.	$11\frac{1}{4}$.
4. $m+4\frac{1}{4}=8$.	$3\frac{3}{4}$.	9. $z+1.25=2.35$.	1.1.
5. $t+2.5=5.5$.	3.	10. $b+1\frac{1}{4}=2.35$.	1.1.

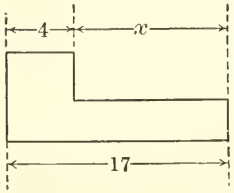

Fig. 3.

Equations may be used in solving other problems than those of the balance.

Example 1. Let $x=$ the missing dimension, Fig. 3,

$$x+4=17$$
$$x=13$$

Example 2. What number added to 14 equals 30?

Let x = the number

$14+x=30$

$x=16$

EXERCISE 2

Find the missing dimensions, using equations:

1

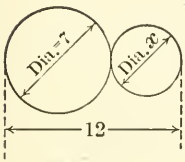

Fig. 4.

Ans. 5.

2

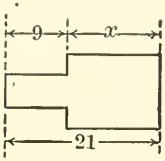

Fig. 5.

Ans. 12.

3

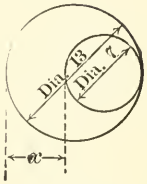

Fig. 6.

Ans. 6.

4. A vacant lot and \$192 were exchanged for another lot worth \$1000. Find the value of the first lot.

Ans. \$808.

3. Checking an Equation. In the equation $3a+5=32$, Fig. 7, if 5 is subtracted from both members, the equation, will then be,

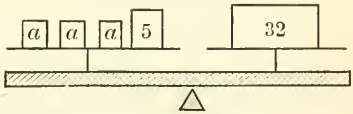

Fig. 7.

$$3a=27.$$

The value of the unknown (a) may then be found by dividing both members by 3, giving,

$$a=9.$$

The problem may be proved by substituting 9 in place of (a) in the original equation. If both members reduce

to the same number, the solution is correct. A complete solution is,

$$3a+5=32$$
$$3a=27 \quad \text{(subtracting 5 from both members)}$$
$$a=9 \quad \text{(dividing both members by 3)}$$

Check. $3\times 9+5=32$
$27+5=32$
$32=32$

EXERCISE 3

Solve and check:	Ans.		Ans.
1. $2a+12=26$.	7.	5. $28t+14=128$.	$4\frac{1}{14}$.
2. $9y+15=96$.	9.	6. $71m+.55=9.07$.	.12.
3. $12m+8=98$.	7.5.	7. $11x+\frac{1}{4}=\frac{89}{4}$.	2.
4. $8+9x=116$.	12.	8. $7w+5\frac{2}{3}=12\frac{2}{3}$.	1.

Find the missing dimensions:

9

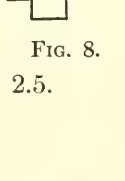

Fig. 8.

Ans. 2.5.

10

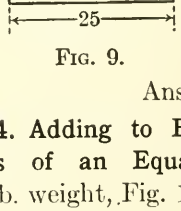

Fig. 9.

Ans. 6.

Fig. 10.

4. Adding to Both Members of an Equation. The 12-lb. weight, Fig. 10, is an upward pull on the pan containing the weight (w). If the scales balance and the 12-lb. weight is removed, a weight

of 12 lbs. must be placed on the other pan to balance the scales. Removing the upward pulling weight is the same as adding the weight to the other pan. Thus, expressed as an equation,

$$w-12=25$$
$$w=37 \text{ (adding 12 to both members)}$$

Example. $5x-11=24$
$$5x=35 \text{ (adding 11 to both members)}$$
$$x=7 \text{ (dividing both members by 5)}$$

Check. $5\times7-11=24$
$$35-11=24$$
$$24=24$$

EXERCISE 4

Solve and check:

1. $x-7=10$. Ans. 17.
2. $2x-11=13$. 12.
3. $12s-34=26$. 5.
4. $2.1t-3.2=3.1$. 3.
5. $3x-9\frac{1}{2}=8.5$. Ans. 6.
6. $17r-3\frac{1}{5}=13\frac{4}{5}$. 1.
7. $4x+11=17$. 1.5.
8. $4x-11=17$. 7.

9. Find the unknown weight in Fig. 11, if the scales balance as shown. Ans. $10\frac{2}{3}$.

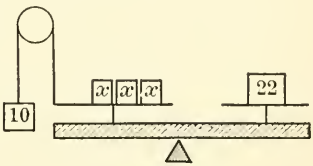

FIG. 11.

10. Three times a number less 26 is equal to 73. Find the number. Ans. 33.

5. Equations with Fractions.

Example. $\frac{1}{5}x=10$
$$x=50 \text{ (multiplying both members by the least common denominator, 5)}$$

Check. $\frac{1}{5}\times50=10$

From this example, it can be seen that both members of an equation may be multiplied by the same number. This method is used in equations where fractions occur.

Example. $\frac{1}{3}x+\frac{2}{5}=3\frac{2}{5}$.

$5x+6=51$ (multiplying both members by the L. C. D. 15)

$5x=45$ (subtracting 6 from both members)

$x=9$ (dividing both members by 5)

Check. $\frac{1}{3}\times 9+\frac{2}{5}=3\frac{2}{5}$

$3\frac{2}{5}=3\frac{2}{5}$

EXERCISE 5

Solve and check:

1. $\frac{1}{2}x-\frac{2}{3}=\frac{1}{3}$. Ans. 2.
2. $\frac{2}{5}x+3\frac{1}{5}=9\frac{1}{5}$. 15.
3. $\frac{y}{3}-12=16$ $\left(\frac{y}{3}\text{ means }\frac{1}{3}y\right)$. Ans. 84.
4. $\frac{1}{2}x-3\frac{1}{2}=7$. Ans. 21.
5. $\frac{3x}{7}-\frac{1}{3}=5\frac{2}{3}$. 14.

6. To Solve an Equation. Observe that:

I. The same number may be added to both members.

$x-3=5$

$x=8$

II. The same number may be subtracted from both members.

$x+7=12$

$x=5$

III. Both members of an equation may be multiplied by the same number.

$\frac{x}{2}=7$

$x=14$

IV. Both members of an equation may be divided by the same number.

$$3x = 15$$
$$x = 5$$

NOTE. In $x-5=8$
$x=8+5=13$
and $x+7=12$
$x=12-7=5$

The effect of adding or subtracting the same number from both members of the equation may be gained by transferring (transposing) the number to the other member and changing the sign before it.

V. An equation may be checked by substituting in the original equation, the value found for the unknown. If both members then reduce to the same number, the solution is correct.

VI. In an equation, such as $2x+5=x+9$, $2x$, 5, x, *and* 9 *are the* **terms** *of the equation,* $2x+5$ *and* $x+9$ *are the* **members** *of the equation separated by the sign of equality* $(=)$.

Example 1. A man saved a certain amount in one year, twice as much in the second year and three times as much in the third year as in the first. In all he saved \$1200. How much did he save during each year?

Let x = amount saved first year
then $2x$ = amount saved second year
and $3x$ = amount saved third year
$x+2x+3x=1200$ (x means $1x$)
$6x=1200$ (combining like terms)
$x=200$
$2x=400$
$3x=600$
1200

Example 2. $x+7x-3x=55.$

$$5x=55 \text{ (combining like terms)}$$
$$x=11$$

Check. $11+77-33=55$

Example 3. $7x-3x+x=55.$

$$5x=55 \text{ (combining like terms)}$$
$$x=11$$

Check. $77-33+11=55$

Example 4. $x-3x+7x=55.$

$$5x=55 \text{ (combining like terms)}$$
$$x=11$$

Check. $11-33+77=55$

NOTE. Observe that in combining $x-3x+7x$, x and $7x$ were first added and the $3x$ then subtracted. In combining $11-33+77$, 11 and 77 were added and the 33 then subtracted. The sign before each term belongs to that term. (+) is understood before the first term if no sign is expressed.

EXERCISE 6

Solve and check:

	Ans.		Ans.
1. $x+11x=24.$	2.	4. $3x-7x+10x=36.$	6.
2. $7m+3m=110.$	11.	5. $x-11x+12x=32.$	16.
3. $17t-3t+t=45.$	3.	6. $2x-7+3x=28.$	7.

Solution.

$$2x-7+3x=28$$
$$5x-7=28$$
$$5x=35$$
$$x=7$$

Check. $14-7+21=28$

NOTE. Observe that the 7 cannot be combined with the $3x$ and $2x$ as it is not a "like term."

7. $13m-2m-12=21$. Ans. 3.
8. $4x+7+3x=91$. 12.
9. $8x-14x+9+7x=37$. 28.
10. $8y-19-17y+12y=11$. 10.

11. The sum of three numbers is 32. The second is twice the first and the third is four times the first. Find the numbers. Ans. $4\frac{4}{7}$, $9\frac{1}{7}$, $18\frac{2}{7}$.

12. A man saved \$800 in three years. The second year he saved three times as much as the first, and the third year he saved \$200. How much did he save the first year? Ans. \$150.

13. A man spent \$175 in January. He paid a certain sum for room rent, twice as much for board, as much for clothing as for board, and \$50 for incidentals. How much did he spend for each? Ans. \$25, \$50, \$50.

14. The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a number is equal to 52. Find the number. Ans. 48.

7. Equations with the Unknown Term in Both Members.

Example. $4x=2x+14$.
$2x=14$ (subtracting $2x$ from both members)
$x=7$

Check. $28=14+14$

Example 2. $7x=48-5x$.
$12x=48$ (adding $5x$ to both members)
$x=4$

Check. $28=48-20$

Example 3. $4m-32=2m$.
$4m=2m+32$ (adding 32 to both members)
$2m=32$ (subtracting $2m$ from both members)
$m=16$

Check. $64-32=32$

NOTE. Observe that terms containing the unknown may be added to or subtracted from both members.

EXERCISE 7

Solve and check:

1. $3t-7=2t+5$. Ans. 12.
2. $13m-42=6+m$. 4.
3. $2y+8+3y+7=18+4y$. 3.
4. $3x-4=x$. 2.
5. $3x-4=17$. 7.
6. $\frac{2}{3}x-2\frac{1}{2}=7\frac{5}{6}-2x$ (clear of fractions). $3\frac{7}{8}$.
7. $4m+12=6m$

   $12=2m$ (subtracting $4m$ from both members)

   $m=6$

Check. $24+12=36$

NOTE. Observe that the unknown may be collected on either side of the sign of equality.

8. $3y+18=9y+6$. Ans. 2.
9. $2y-2\frac{1}{4}=5y-17\frac{1}{4}$. 5.
10. $17=2x-3$. 10.
11. $\frac{3n}{2}+40=\frac{6}{7}n-7+4n$. 14.
12. $1x+62=3x+2$. 30.
13. $\frac{x}{5}+2=\frac{2x}{5}-1\frac{4}{5}$. 19.
14. $7x+20-3x=60+4x-50+8x$. $1\frac{1}{4}$.
15. $3m+60=15m+3-2m+7$. 5.

8. Equations with the Unknown in the Denominator. Equations sometimes occur in which the unknown appears in the denominator. In such equations, the unknown becomes part of the common denominator.

EXERCISE 8

Solve and check:

1. $\frac{21}{2a}=\frac{3}{4}$. Ans. 14.

   $42=3a$ (multiplying both members by the L. C. D. $4a$)

2. $\frac{13}{2x}+\frac{5}{3x}=1\frac{1}{6}$. 7.

   $39+10=7x$ (multiplying both members by the L. C. D. $6x$)

3. $\frac{11}{a}=3\frac{2}{3}$. 3.

4. $\frac{12}{m}-\frac{4}{m}=\frac{2}{3}$. 12.

5. $\frac{15}{2x}+\frac{25}{4x}=5\frac{1}{2}$. $2\frac{1}{2}$.

EXERCISE 9

Review

Solve and check:

1. $2x-9=27$. Ans. 18.
2. $12x-9x=14-4x$. 2.
3. $15-3x+11x=39$. 3.
4. $\frac{8}{5}-\frac{t}{3}=\frac{t}{5}-3\frac{1}{10}+\frac{t}{4}$. 6.

Write equations and solve:

5.

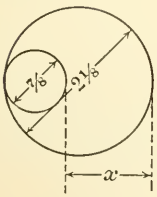

Fig. 12.

Ans. $1\frac{1}{4}$.

6.

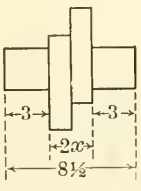

Fig. 13.

Ans. $1\frac{1}{4}$.

7.

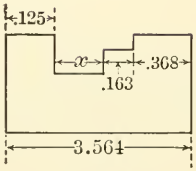

Fig. 14.

Ans. 2.908.

8. Find the diameters of the three circles, Fig. 15, if the diameter of the second is twice that of the first and of the third $1\frac{1}{2}$ that of the first. Ans. $3\frac{1}{3}$, $6\frac{2}{3}$, 5.

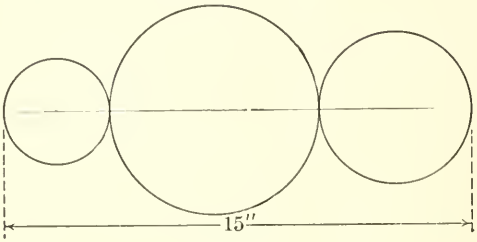

Fig. 15.

9. Three men together received \$335. The first received \$11 more than the second and \$16 less than the third. Find what each received. Ans. \$99, \$110, \$126.

10. One man has three times as many acres of land as another. After selling 60 acres to the second man, he has still 40 acres more than the other. How many acres had each at first? Ans. 80, 240.

CHAPTER II

SLIDE RULE

THE following rules and directions for the use of the slide rule are not intended for a complete manual of instructions but are intended to present in a simple readable manner instructions for solving the problems ordinarily solved on the slide rule. Practice problems are furnished so that the student will have had some practice before he begins to use the slide rule in a practical way. Theory of the rule will not be discussed, but considerable attention will be paid to the principles of operation which make for efficient handling of the rule. The work on the location of the decimal point may be omitted if desired.

9. Common Types of Slide Rules:

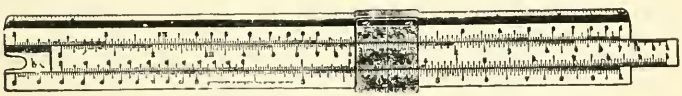

FIG. 16.—Mannheim Slide Rule. Recommended for general use.

FIG. 17.—Polyphase Slide Rule. Recommended for general use and in some problems is more efficient than the Mannheim rule.

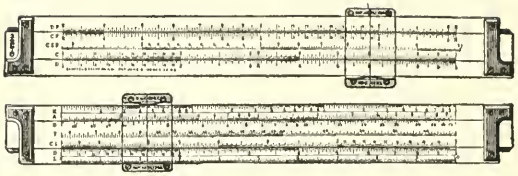

FIG. 18.—Polyphase Duplex Slide Rule. Useful in problems involving π (=3.1416) but not recommended for general use.

10. Scales. The face of most slide rules has four scales, the *A*, *B*, *C* and *D* scales. The *A* scale is on the upper part of the rule, the *B* scale on the upper part of the slide, the *C* scale on the lower part of the slide and the *D* scale on the lower part of the rule. The polyphase duplex has the *C* and *D* scales but not the *A* and *B*, the other two types have the four scales.

The *C* and *D* scales are the ones in most common use and so will be considered first. Notice on your rule that the *C* and *D* scales are alike. Study the following relations on the *D* scale by reference to your rule. The scale as a whole is divided into 10 parts marked 1, 2, 3, 4, 5, 6, 7, 8 , 9, 1, as in Fig. 19.

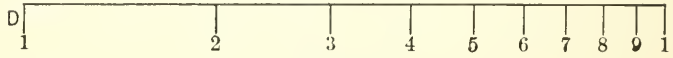

FIG. 19.

These divisions correspond to the numbers 1, 2, 3, 4, etc., but the numbers on the slide rule have no decimal points, so the 3 stands for 3, 30, 300, 3000, etc., and the 4 for 4, 40, 400, etc. Numbers between 1 and 2, 2 and 3, etc., must be read on the unnumbered divisions between the numbered ones or sometimes approximated between the

small division lines. Each interval 1–2, 2–3, 3–4, etc., is divided into 10 main subdivisions indicated by the longest lines between the numbers. The main subdivisions between 1 and 2 are numbered 1, 2, 3, −9; but they are not so numbered on the rest of the rule. Great care must be taken not to confuse these numbers with the numbers on the main part of the scale. Each main division being divided into 10 main subdivisions, these main subdivisions will stand for 11, 12, 13, . . . , 21, 22, 23, 24, . . . , 31, 32, 33, . . . , 41, 42, etc.

EXERCISE 1

Locate the following numbers on the *D* scale and have the instructor verify three or four of your answers:

1. 5.
2. 7.
3. 65.
4. 68.
5. 23.
6. 50.
7. 55.
8. 20.
9. 19.
10. 15.
11. 18.
12. 92.

The main subdivisions of a slide rule are again subdivided into ultimate subdivisions. Due to the different lengths of the divisions some of the main subdivisions are divided into 10 spaces, some into 5 spaces, and others into 2 spaces. These ultimate subdivisions are the divisions between 23 and 24, . . . 77 and 78, etc., hence they correspond to the third figure of a number. For example, find 15 and 16 on the *D* scale. The divisions between 15 and 16 stand for 151, 152, 153, 154, 155, 156, 157, 158, 159.

EXERCISE 2

Find the following numbers of the *D* scale and have the instructor verify three or four of the answers:

1. 125.
2. 146.
3. 181.
4. 195.
5. 175.
6. 115.
7. 105.
8. 103.
9. 108.
10. 199.

Between 2 and 4 the main subdivisions are all divided into only five ultimate subdivisions, hence each ultimate subdivision stands for 2 in the third figure of a number. For example find 22 and 23 on the *D* scale. The divisions between 22 and 23 stand for 222, 224, 226, 228.

EXERCISE 3

Find the following numbers on the *D* scale and have the instructor verify three or four of the answers:

1. 232.
2. 254.
3. 212.
4. 210.
5. 208.
6. 332.
7. 368.
8. 298.
9. 108.
10. 198.

Between 4 and the end of the rule, the main subdivisions are divided into only two ultimate subdivisions, hence each ultimate subdivision stands for 5 in the third figure of a number.

EXERCISE 4

Find the following numbers on the D scale:

1. 435.
2. 565.
3. 670.
4. 675.
5. 885.
6. 863.
7. 995.
8. 715.
9. 710.
10. 705.

11. Numbers Not Represented by a Line. Find 214 and 216 on the D scale; then 215 is midway between these two marks. To find 2155, set the cross mark of the runner $\frac{3}{4}$ of the way between 214 and 216. Find 420 and 425 on the D scale; then 422 would be $\frac{2}{5}$ of the way between these two marks. Reading the numbers between the lines on a slide rule is not exact and becomes a matter of judgment and experience.

EXERCISE 5

Find the following numbers on the D scale:

1. 225.
2. 365.
3. 485.
4. 477.
5. 585.
6. 518.
7. 755.
8. 757.
9. 25.
10. 125.
11. 175.
12. 1625.
13. 115.
14. 1125.
15. 106.
16. 1065.

EXERCISE 6

Read the number represented by the positions of Fig. 20.

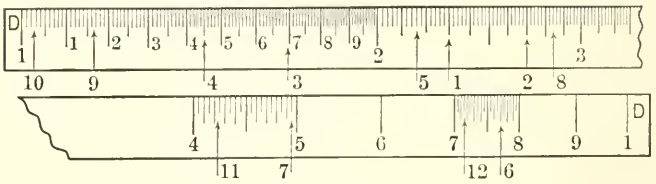

Fig. 20.

1.	Ans. 23.	7.	Ans. 495.
2.	27.	8.	284.
3.	17.	9.	1165.
4.	145.	10.	103.
5.	216.	11.	422.
6.	77.	12.	715.

12. Division. To divide 6 by 3, find 6 on the *D* scale and move the slide to the right until the 3 on the *C* scale is directly above the 6. Find the answer on the *D* scale under the end of the *C* scale. Try this setting on your rule. To divide 20 by 5, find the 20 on the *D* scale and move the slide until the 5 on the *C* scale is directly above the 20 on the *D* scale. The answer is on the *D* scale under the end of the *C* scale.

Rule. *To divide two numbers find the dividend on the D scale and set the divisor (on the C scale) directly above the dividend. The quotient will be found on the D scale under the end of the C scale.*

EXERCISE 7

Divide:

1. $6 \div 3$.	Ans. 2.	5. $168 \div 2$.	Ans.	84.
2. $20 \div 4$.	5.	6. 256×7.		36.6.
3. $24 \div 8$.	3.	7. $653 \div 21$.		31.1.
4. $65 \div 5$.	13.	8. $768 \div 125$.		6.14.

13. Multiplication. Multiplication, in arithmetic, is the opposite of division, hence the operation of multiplication on the slide rule is the opposite of division on the slide rule. In arithmetic the check for division is the quotient times the divisor should equal the dividend. Thus:

$$6 \div 3 = 2.$$

Proof. $$6 = 3 \times 2.$$

Therefore to multiply 2×3 on the slide rule the 2 will be in the same position as the 2 in $6 \div 3$; that is, on the *D* scale under the end of the *C* scale. 3 is found on the *C* scale and the answer on the *D* scale under the 3. Try this on your rule. Try also 25×3.

Rule. *To multiply two numbers, place the end of the C scale on one of the numbers (on the D scale) and find the answer on the D scale under the other number (on the C scale).*

In division the answer was under one end of the *C* scale in some problems and under the other end in other problems, and so in multiplication one end of the *C* scale is used in some problems and the other end in other problems. Use the end which will bring the other number above the *D* scale. For example, in 4×5 place the left end of the *C* scale on 4 and look for 5 on the *C* scale, it comes beyond the end of the *D* scale, therefore the right end of the *C* scale must be set on the 4 and under 5 will be found the answer 20.

EXERCISE 8

1. 2×3.	Ans.	6.	6. 75×22.	Ans.	1650.
2. 4×12.		48.	7. 125×55.		6775.
3. 18×5.		90.	8. 223×64.		14280.
4. 5×6.		30.	9. 175×235.		41100.
5. 7×4.		28.	10. 1575×45.		70800.

14. Location of the Decimal Point. Heretofore the location of the decimal point has been left to the student to determine by estimating how large the answer should be. In practice the decimal point is usually located in this manner, but it is well to know the rules.

Rule. *In multiplication, if the slide projects to the left, add the number of places in both factors to determine the number of places in the product. If the slide projects to the right, add the number of places in both factors and subtract 1 to determine the number of places in the product. In division if the slide projects to the left, subtract the number of places in the divisor from the number of places in the dividend to determine the number of places in the quotient. If the slide projects to the right, subtract the number of places in the divisor from the number of places in the dividend and add 1 to determine the number of places in the quotient.*

The following diagram should prove helpful in memorizing the above rules:

	Slide projects to the Left	Right
Multiplication	Add	Add, subtract 1
Division	Subtract	Subtract, add 1

In the above rules do not count the figures in the decimal part of a mixed number. In the case of decimals only the number of places is considered negative and is

equal to the number of zeros between the decimal point and the first significant figure.

EXERCISE 9

Solve the following problems paying attention to the location of the decimal point.

1.	$18 \div 12$.	Ans. 1.5.
2.	16×8.	128.
3.	$18 \div 3$.	6.
4.	16×2.4.	38.4.
5.	537×38.	20400.
6.	1250×6.25.	7820.
7.	2500×890.	2225000.
8.	234×0.012.	2.808.
9.	5620×3.45.	19400.
10.	$25 \div 0.0062$.	0.155.

15. Continued Multiplication. In the use of the slide rule great care must be taken to develop efficiency or the rule will never become a labor-saving device. In finding the product of 3 factors, as $12 \times 14 \times 24$, speed and accuracy can be gained by noting that, after 12 is multiplied by 14 in the usual way, the answer appears on the D scale. It will not be necessary to read this number as it is in place for multiplying it by 24; therefore, mark this product with the crossmark of the runner and then multiply the number represented by this position by 24 by drawing the end of the C scale under the crossmark and find the product on the D scale below the 24 on the C scale. Try this on your rule. Marking the first product in this way saves time and eliminates the error of reading this product and again setting to the number read.

EXERCISE 10

1. $2\times3\times4$.	Ans.	24.
2. $12\times7\times13$.		1092.
3. $250\times3\times78$.		58500.
4. $172\times305\times450$.		23600000.
5. $751\times0.046\times231$.		7980.

16. Formulas of the Type $R=\frac{Kl}{A}$. In problems of this type it is most efficient to first divide K by A and then multiply by l. For example, $R=\frac{25\times48}{32}$ Divide 25 by 32 in the usual way; the result appears on the D scale under the end of the C scale. Then to multiply this number by 48, find 48 on the C scale and the answer, 375, appears on the D scale under the 48 on the C scale. The student should keep in mind the fact that, in problems of this type, it is best to do the division first, since this leaves the rule set for multiplying without moving the slide. All problems of this type can be done with one setting of the rule; but in some cases the second factor will appear off the scale. It is then necessary to change ends with the slide before doing the multiplication. This is accomplished by marking the end of the C scale with the crossmark of the runner and bringing the other end of the C scale under the runner. Changing ends of the slide is not called a slide rule operation.

EXERCISE 11

1. $\frac{5\times12}{4}$.	Ans.	15.
2. $\frac{17.5\times32}{15}$.		37.3.

3. $\frac{225\times13.2}{790}$. 3.76.

4. $\frac{25\times3.3\times21}{8\times17}$. 127.4.

17. Proportion. Set on your rule 2 above 3, and notice that any other pair of corresponding numbers are in the ratio of 2 to 3. For example, 4 is above 6 and 12 is above 18, etc. For any setting of the slide any two pairs of corresponding numbers are in the same ratio. This makes a simple method for solving proportion.

For example,

$$\frac{16}{27}=\frac{x}{64}.$$

Place 16 above 27 and find x above 64.

In some problems it will be necessary to change ends with the slide after first setting for the one ratio.

Observe that:

When in any problem a number appears off scale, it is necessary to change ends with the slide.

EXERCISE 12

1. $\frac{2}{3}=\frac{x}{9}$. Ans. 6.

2. $\frac{3}{5}=\frac{18}{x}$. 30.

3. $\frac{125}{x}=\frac{371}{255}$. 85.8.

4. $\frac{x}{289}=\frac{350}{1250}$. 80.8.

5. $\frac{24.5}{38.1}=\frac{7500}{x}$. 11690.

18. The A and B Scales. Most slide rules have an A scale on the upper part of the rule and a B scale on the upper part of the slide. Make sure from the instructor or otherwise, that your rule has these two scales before proceeding with the following directions:

The A and B scales are the same and are composed each of two slide rule scales placed end on end. The two scales will be distinguished as the right scale and left scale. Numbers are read as follows: On the left scale 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and on the right scale 10, 20, 30, 40, 50, 60, 70, 80, 90, 100; 100, 200, 300, etc., appear again on the left scale and 1000, 2000, 3000, appear on the right scale. Or, in other words, the 4 on the left scale stands for 4, 400, 40000, etc., while the 4 on the right scale stands for 40, 4000, etc., and not for 4 or 400, etc.

Rule. *Numbers having an odd number of places are found on the left scale, and numbers having an even number of places are found on the right scale.*

In counting the number of places in a number do not count the figures in the decimal part of a mixed number. In the case of decimals only, the number of places is determined by the number of zeros between the decimal point and the first significant figure. Thus .04 is found on the left scale and .00423 is found on the right scale. A decimal with no zeros before the significant figure is found on the right scale.

The A and B scales being shorter than the C and D scales there are not as many ultimate subdivisions. The student should study the scales immediately to determine the number of and value of the ultimate subdivisions.

19. Use of he A and B Scales. The A and B scales are used in connection with the C and D scales to find squares and square roots and to solve problems involving squares and square root.

Find several numbers of the D scale as 2, 5, 12, 8, 34, and notice that the squares of these numbers 4, 25, 144, 64, 1156, appear on the A scale directly above the numbers on the D scale. The same relation exists between the B scale and C scale. To find the square root of a number, find the number on the A or B scale, and find the square root on the D or C scale directly below the number.

EXERCISE 13

Find the square root of the following numbers:

1. 4.	Ans. 2.	4. 563.	Ans. 23.7.
2. 36.	6.	5. 1582.	39.8.
3. 144.	12.	6. 75000.	274.

In handling formulas involving squares and square roots do the multiplications and divisions on the A and B scales, using the C and D scales only to locate such numbers as 17^2, 352^2, etc., or to find the square root of a product or quotient. The rules for multiplication and division on the A and B scales are the same as for multiplication and division on the C and D scales.

Example 1. $7\times5^2=?$ Find 7 on the A scale and set the right end of the B scale under it the answer will appear on the A scale above 5^2 on the B scale, that is, above the 5 on the C scale; use the crossmark on the runner. It is not necessary to read the value of 5^2.

Example 2. $\frac{36}{2^2}=?$ Find 36 on the A scale place 2^2 on the B scale under it, using the 2 on the C scale to locate the 2^2 on the B scale. The answer is on the A scale above the end of the B scale.

Example 3. $\frac{12^2}{18}=?$ Find 12^2 on the A scale (above

12 on the D scale); set 18 on the B scale below 12^2. The answer appears on the A scale.

Example 4. $\sqrt{\frac{5}{17}\frac{3}{}}=?$ Or $\sqrt{23\times 7}=?$ Do the multiplication or division on the A and B scales and the answer will be found on the C or D scale below the result of multiplication or division on the B or A scale. It is not necessary to read the result on the A or B scale.

Example 5. $\frac{12\times 18}{5^2}=?$ This is similar to the $\frac{Kl}{A}$ formula, and so 12 should be divided by 5^2 first, as in Example 2. This leaves the rule set for multiplying by 18 without resetting the slide.

20. General Suggestions. The following suggestions should prove helpful in the use of the slide rule.

1. *The A and B scales are used in square and square root formulas, the C and D scales being brought into use to locate such numbers as 27^2, etc., or to find a square root.*

2. *The principles of operation of the A and B scales are the same as for the C and D scales.*

3. *In division the dividend may be located on either scale and the divisor on the opposite scale, but the quotient will always appear on the scale with the dividend.*

4. *In formulas involving more than one operation keep in mind where the result would appear after each operation. It is not necessary to read these intermediate results.*

5. *Division leaves the rule set for multiplication without moving the slide.*

21. Areas of Circles. Finding the area of circles is similar to Example 1 above, but it will be noticed that π ($=3.1416$) is marked with a special mark on the A and B scales. To find the areas of a circle place the end of the B scale on π on the A scale and find the areas on the A scale above the radius square of the B scale (that is, above the radius on

the C scale). With this one setting the areas of any number of circles can be read off on the A scale above the radius on the C scale.

22. Ratios of the Same Kind. Different values can be read from one setting of the rule in many other problems. For example, one setting makes it possible to read off the number of centimeters corresponding to any number of inches, or to read the number of pounds corresponding to any number of kilograms, or many other ratios based on the same constant. A table of these various settings and proportions will be found on the back of most single face rules and in the slide rule manual.

EXERCISE 14

REVIEW

1. 27×83.	Ans. 2241.
2. $6\times17\times35$.	3570.
3. $167\div14$.	11.93.
4. $\frac{27\times35}{13}$.	72.7.
5. $\frac{16\times83\times41}{125\times7}$.	62.2.
6. 21^2.	441.
7. $\sqrt{134}$.	11.58.
8. $32^2\times5$.	5120.
9. 18×17^2.	5200.
10. $(9\times64)^2$.	332000.
11. $\sqrt{35\times72}$.	50.2.
12. $\sqrt{\frac{139}{22}}$.	2.514.
13. $\frac{27\times22}{17^2}$.	2.06.

14. $\frac{28^2}{11}$. 71.3.

15. $\frac{1230}{24^2}$. 2.14.

16. $\sqrt{24 \times 169}$. 63.7.

17. Find the areas of the circles having for radius (1 setting):

(*a*) 3 in.	Ans.	28.3.
(*b*) 14 in.		616.
(*c*) 37 in.		4300.
(*d*) 2.68 in.		22.6.

18. Change to centimeters the following inches (1 setting):

(*a*) 8.	Ans.	20.3.
(*b*) 67.		170.
(*c*) 12.3.		31.2.
(*d*) 2.26.		5.74.

19. Change to inches the following centimeters:

(*a*) 27.	Ans.	10.63.
(*b*) 19.		7.48.
(*c*) 184.		72.4.
(*d*) 643.		253.1.

The work of this chapter is not a complete slide rule manual, but the student should be able to do the problems ordinarily done on the slide rule rapidly and efficiently. He needs still more practice to become accurate, rapid, and confident. There is still much to be learned on the slide rule, and the student should begin to study the manual furnished with slide rules, and to watch his own work to see where he can increase his speed and efficiency.

CHAPTER III

EVALUATION

23. General Numbers. The area of a rectangle is found by multiplying the base by the altitude. This may be expressed by $(b)\times(a)$, in which the value of (b) may be 12 ft., 7 in., 25 rods, or any number of any unit used to measure length, and (a) may be any number of a like unit. Letters which may represent different values in different problems are called *general numbers.* The unknown letters used in Chapter I are general numbers.

EXERCISE 1

1. Find $b+a$ when $b=3$, $a=7$. Ans. 10.
2. Find $b+a$ when $b=5$, $a=12$. 17.
3. Find $\frac{m}{n}+\frac{x}{y}$ when $m=3$, $n=4$, $x=5$, $y=8$. $1\frac{3}{8}$.
4. Find $R-\frac{n}{d}$ when $R=5$, $n=4$, $d=8$. $4\frac{1}{2}$.

24. Signs. When the multiplication of two or more factors is to be indicated, the sign of multiplication is often omitted or expressed by the sign $(\cdot)$; thus $7\times a\times b\times m$ is written $7\cdot a\cdot b\cdot m$, or more often $7abm$. Care must be taken in the use of the sign $(\cdot)$ to distinguish it from the decimal point, $7\cdot 9$ means 7×9, while 7.9 means $7\frac{9}{10}$.

EXERCISE 2

1. Find ax when $a=3$, $x=5$. Ans. 15.
2. Find $3mn$ when $m=2$, $n=7$. 42.

NOTE. When addition, subtraction, multiplication and division occur in the same problem, do the multiplication and division first in the order in which they occur, then do the addition and subtraction.

3. Find $ax+by$ when $a=4$, $x=7$, $b=3$, $y=4$. Ans. 40.
4. Find $2ad-3cd$ when $a=12$, $d=4$, $c=5$. 36.

25. Coefficient. When the multiplication of two factors is expressed as ax, either factor is called the *coefficient* of the other.

EXERCISE 3

What is the coefficient of x in the following:

1. bx.
2. $5x$.
3. $3ax$.
4. $(a+b)x$.
5. $(2d-3)x$.
6. xy.
7. $x(m-n)$.

26. Power. If all the factors of a product are the same, the product is called a *power* of that factor. Thus $xxxx$ is x fourth power and is written x^4. Similarly,

$xx = x^2$, and is read x 2d power or x square.
$xxx = x^3$, and is read x 3d power or x cube.
$xxxx = x^4$, and is read x 4th power.
$xxxxx = x^5$, and is read x 5th power.

27. Exponent. The number which indicates the power is written as a small number above and to the right of the factor and is called its *exponent*. For example, in x^2, 2 is the exponent, in y^3, 3 is the exponent.

28. Base. The number which is used as a factor a number of times is called the *base*. For example, in x^3, x is the base and 3 is the exponent; in 5^3, 5 is the base, 3 is the exponent and the power is equal to 125.

EXERCISE 4

1. Find x^2	when $x=5$.	Ans.	25.
2. Find m^3	when $m=2$.		8.
3. Find x^4-y^2	when $x=3,\ y=5$.		56.
4. Find a^2b^3	when $a=4,\ b=2$.		128.
5. Find ax^3	when $a=6,\ x=4$.		384.
6. Find $5x^3$	when $x=4$.		320.

29. Signs of Grouping. The sign of grouping most commonly used is the parenthesis ( ). It means that the parts enclosed are to be taken as a single quantity. For example: $3(x+y)$ means 3 times the sum of x and y; $(x+y)^3$ means $(x+y)(x+y)(x+y)$.

EXERCISE 5

1. Find $3(15-7)$.		Ans.	24.
2. Find $3(x+y)$	when $x=2,\ y=4$.		18.
3. Find $2(x^2-a^3)$	when $x=7,\ a=3$.		44.
4. Find $5(2x+3ay)$	when $x=4,\ a=2,\ y=6$.		220.
5. Find $5x(3m^2-2n^2)$	when $x=2, m=8,\ n=5$.		1420.

30. Evaluation. Evaluation of an expression is the process of finding its value by substituting definite numbers (figures) for general numbers and performing the operations indicated.

EXERCISE 6

Evaluate:

1. $3a^2x^3$ when $a=3,\ x=2$.

Solution.

$$\begin{aligned} &3\ a^2\ x^3 \\ &3\cdot 3^2\cdot 2^3 \\ &3\cdot 9\cdot 8 \\ &216. \quad \text{Ans.} \end{aligned}$$

2. $2x(a^3-y)$ when $x=5,\ a=4,\ y=30$.

Solution.

$$\begin{aligned} &2x(a^3-y) \\ &2\cdot 5(4^3-30) \\ &10(64-30) \\ &10\cdot 34 \\ &340. \quad \text{Ans.} \end{aligned}$$

3. $2x$ when $x=5$. Ans. 10.

4. x^2 when $x=5$. 25.

5. $3(x+y)$ when $x=2,\ y=4$. 18.

6. $4(a+b)^2$ when $a=2,\ b=4$. 144.

7. x^2y^3 when $x=3,\ y=2$. 72.

8. $7x^2$ when $x=4$. 112.

9. x^3-a^2 when $x=5,\ a=3$. 116.

10. $2a^2x^3(a-b)$ when $a=12,\ x=2,\ b=8$. 9216.

11. $\frac{1}{x}+\frac{5}{n}+\frac{3}{R}$ when $x=2,\ n=8,$ $R=16$. $1\frac{5}{16}$.

31. Formulas. A formula is the statement of a rule or principle in terms of general numbers. For example, the area of a rectangle is equal to the product of the base by the altitude. Stated as a formula this becomes $A=ba$.

32. Evaluation of Formulas. Evaluation of a formula is the process of substituting definite numbers for general

numbers, and solving the resulting equation for the one remaining general number. For example, evaluate:

$$A = \tfrac{1}{2}bh \text{ when } b = 5,\ h = 12.$$

Substituting values,

$$A = \tfrac{1}{2} \cdot 5 \cdot 12$$

Whence,

$$A = 30$$

Evaluate,

$$x + 3r = 5a \text{ when } r = 2,\ a = 7$$

Substituting values,

$$x + 3 \cdot 2 = 5 \cdot 7$$

Multiplying,

$$x + 6 = 35$$

Whence,

$$x = 29$$

33. Electrical Formulas—Resistances in Series. The total, or combined resistance, R, of the three resistances R_1,

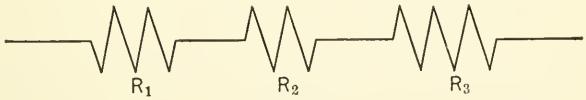

FIG. 21.

R_2 and R_3 in series, Fig. 21, is expressed by the equation

$$R = R_1 + R_2 + R_3.$$

All are measured in ohms.

EXERCISE 7

Evaluate the formula to find the missing values:

	R	R_1	R_2	R_3	
1.		12	5	8	Ans. 25.
2.		6.3	2.9	4.43	13.63.
3.		6.8	4.6	2.01	13.41.
4.	10	2	1		7.
5.	120	6	70		44.
6.	16		3	4	9.
7.	25	12.5		2.6	9.9.
8.	19.1	14		4.41	0.69.
9.	20		6.4	9.98	3.62.

10. The combined resistance of four lamps in series is 1025 ohms. The first lamp has a resistance of 250 ohms, the second, 260 ohms and the third 290 ohms. What is the resistance of the fourth lamp? Ans. 225.

11. Find the total resistance of three coils in series if the first has a resistance of 2 ohms, the second 5 ohms, and the third 6 ohms. Ans. 13.

12. Four storage batteries are connected in series for charging, 3 have internal resistances of 4 ohms each and the fourth has an internal resistance of 6 ohms. How many ohms must be put in series to make a total resistance of 25 ohms? Ans. 7.

34. Ohm's Law. Ohm's law for the relation of voltage to current and resistance in a circuit is expressed by the formula:

$$E = IR$$

where, E = voltage,
I = current measured in amperes,
R = resistance measured in ohms.

EXERCISE 8

Evaluate the formula to find the missing factors:

	E	I	R		
1.		.55	11	Ans.	6.05.
2.		.75	9		6.75.
3.	110		220		.5.
4.	110	.75			146.7.
5.	220		450		.488.
6.	6	1.5			4.

7. If $I=\frac{E}{R}$, find I when $E=110$ volts and $R=9$ohms. Ans. 12.2.

8. If $R=\frac{E}{I}$, find R when $E=110$ volts and $I=0.5$ amperes. Ans. 220.

9. How much current will flow through the windings of an electromagnet of 140 ohms resistance, when placed across a 110 volt circuit? Ans. .185.

10. What is the resistance of an incandescent lamp on a 110 volt circuit if the current is 1.5 amperes? Ans. 73.3.

11. Four lamps in series have resistances of 2 ohms, 3 ohms, 2.5 ohms and 3.25 ohms. If 119 volts are applied find the number of amperes current flowing. Ans. 11.07.

12. An electric car heater is supplied with 500 volts from the trolley. The current is 2.5 amperes, what is the resistance? Ans. 200.

13. Find the current sent through a circuit of 200 ohms resistance by a Daniell cell cell which has a voltage of 1.09 volts. Ans. 0.00545.

35. Work Done by an Electric Current. The work done by an electric current equals the power times the time. Expressed as an equation this is $W=PT$. The work is measured in watt-hours or kilowatt-hours, the power in watts or kilowatts, and the time in hours.

EXERCISE 9

Evaluate the formula to find the missing factors:

	W	P	T	
1.	110	55		Ans. 2.
2.		75	7	525.
3.	485		3.25	149.23.

4. What will be the cost at 4 cents per kilowatt-hour to run a 4400 watt heater for 6 hours? (1 kilowatt = 1000 watts.) Ans. $1.06.

5. How much work is done when a 40 watt lamp is lighted for 4 hours? Ans. 160 W.H.

6. The power used in doing a piece of work is 100 kilowatts and the time taken is 3 hours. How much work is done? Ans. 300 K.W.H.

7. The work done by a heater is equal to 50 kilowatt-hours, and the time is 200 min. What is the power?

Ans. 20 K.W.

8. The power used by an arc lamp is 500 kilowatts and the work done is 2500 kilowatt-hours. Find the time.

Ans. 5 Hrs.

36. Heat in an Electric Circuit. The heat developed by an electric current is expressed by the formula:

$$H = 0.24\ Pt,$$

where, H = Heat in calories,*
P = power in watts,
t = time in seconds.

The constant 0.24 is called the *heat equivalent* of electricity.

* A calorie is the amount of heat required to raise the temperature of one gram of water one degree Centigrade.

EXERCISE 10

Evaluate the formula to find the missing factors:

	H	P	t	
1.		89	1.5 hrs.	Ans. 115344.
2.	230000		3 hrs. 20 min.	79.8.
3.	189000	67		3.26.

4. How much heat is generated per hour in an electric iron using 660 watts? Ans. 570240.

37. Electricity Stored in a Condenser. The electricity stored in a condenser is expressed by the equation:

$$Q = CE,$$

where, Q = quantity of electricity measured in coulumbs,
C = capacity of the condenser in farads,
E = applied voltage.

EXERCISE 11

1. 45 volts are applied to a condenser of 2 microfarads capacity. How much electricity is stored? (1 microfarad = .000001 farad.) Ans. 0.00009.

2. How much voltage is required to store 0.005 coulumb in a condenser of 250 microfarads capacity? Ans. 20.

38. Transformation of Formulas. Ohm's law may be expressed in any one of the following forms:

1. $E = IR.$
2. $R = \frac{E}{I}.$
3. $I = \frac{E}{R}.$

1 is the most convenient for finding E, 2 for finding

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R, and 3 for finding I. It is not necessary to memorize more than one of these formulas. The other two may be derived from any one as follows:

1. From 1 derive 2:

$$E = IR$$
$$\frac{E}{R} = \frac{IR}{R} \quad \text{(Dividing both members by } R\text{)}$$
$$\frac{E}{R} = I \quad \text{(Cancellation)}$$

2. From 2 derive 1:

$$R = \frac{E}{I}.$$
$$IR = \frac{EI}{I} \quad \text{(Multiplying both members by } I\text{)}$$
$$IR = E \quad \text{(Cancellation)}$$

EXERCISE 12

1. From 1 derive 3.
2. From $Q = CE$ derive formulas for C and E.
3. From $Q = IT$ derive formulas for I and T.

General Directions. To change the form of a formula first clear the formula of fractions. Then get the letter to be solved for on one side of the equation, and all other letters on the other side.

39. Force of Attraction or Repulsion on a Magnet. A magnet brought into a magnetic field is acted upon by a force expressed by the formula:

$$F = MH,$$

where, F = force of attraction or repulsion in dynes,
M = strength of the magnet in unit poles,
H = intensity of magnetic field in gausses.

EXERCISE 13

Evaluate the formula to find the missing terms.

	F	M	H	
1.		600	3000	Ans. 1800000.
2.	4000000		2500	1600.
3.	65000	200		325.

4. A magnet of 600 unit poles is placed in a magnetic field of 5000 gausses intensity. What force is exerted upon the magnet? Ans. 300000 dynes.

5. Solve the formula $F = MH$ for M and for H.

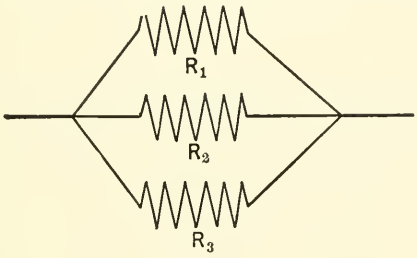


FIG. 22.

40. Resistances in Parallel. The formula

$$R = \frac{1}{\frac{1}{R_1} \times \frac{1}{R_2} \times \frac{1}{R_3}}$$

is used for finding the combined or equivalent resistance of resistances connected in parallel as in Fig. 22. R_1, R_2, and R_3 are the separate resistances, R is the combined resistance. All resistances are measured in ohms.

EXERCISE 14

Evaluate the formula to find the missing terms.

	R	R_1	R_2	R_3	
1.		15	8	12	Ans. 3.64.
2.		220	3	12	2.37.
3.		210	210	210	70.

4. Three lamps of 195 ohms each are connected in parallel. Find the resistance. Ans. 65.

5. Five lamps are connected in series, each has a resistance of 80 ohms. Another series of 4 similar lamps is connected in parallel with the first series. Find the total resistance. Ans. 177.7.

6. If 220 volts are applied to the above circuit, find the current. Ans. 1.237.

41. Computing Resistance of a Conductor from Its Size and Length. The resistance in ohms of a conductor of known material may be computed from the formula,

$$R = \frac{Kl}{A},$$

where, R = resistance of the conductor in ohms,
l = length of the conductor in feet,
A = area of the conductor in circular mils,
K = a constant depending on the material in the conductor, and is equal to the resistance per mil foot in ohms.

One **circular mil** is the area of a circle 1 mil in diameter. A **mil** is one thousandth of an inch.

A circle 2 mils in diameter has an area of 4 circular mils, and a circle 3 mils in diameter has an area of 9 circular

mils. *The area of any circle in circular mils is equal to the square of the diameter in mils.* The circular mil is a convenient unit for measuring the areas of wires since it eliminates the use of π (3.1416).

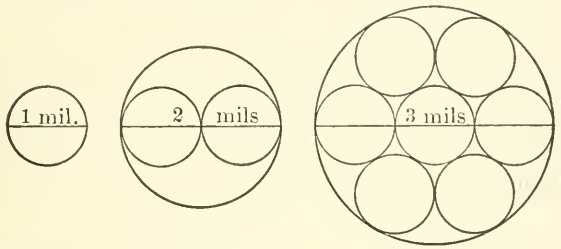


FIG. 23.

EXERCISE 15

Evaluate the formula to find the missing factors:

	R	K	l	A	Ans.
1.		10.4	1000′	16510	.630.
2.		10.4	275′	810	3.525.
3.		10.4	1 mile	6530	8.41.
4.	0.528		25′	509	10.8.
5.	7.79		25	409	127.4.
6.	13.27	10.4		404	515′.
7.	5.5	10.4	850′		1607.
8.		10.4	1 mile	10381	5.29.

9. Find the resistance of 680 feet of No. 25 B. S. gauge German silver wire ($K = 125$, $A = 320.4$). Ans. 265.5.

10. Find the resistance of 20 miles of trolley wire made of No. 00 B. S. gauge copper wire ($K = 10.4$, $A = 133080$). Ans. 8.26.

11. Find the area of cross-section of copper wire having a resistance of 140 ohms per mile. Ans. 392.2.

12. It is desired to transmit 200 amperes to a point 2500 feet from the generator with not more than 4 volts line drop. What diameter copper wire must be used?

Ans. 1140 mils.

13. It is desired to transmit 50 amperes 5 miles with a line drop of not more than 10 volts. What size copper wire must be used? Ans. Area = 1372800 circular mils.

14. Solve the formula $R = \frac{Kl}{A}$ for K and for A.

Ans. $K = \frac{RA}{l}$, $A = \frac{Kl}{R}$.

42. Resistance of a Conductor at Different Temperatures. When the temperature of most conductors changes, their resistances change also. The formula used to find resistance change with change of temperature is

$$R_f = R_i + R_i at,$$

where, R_i = initial resistance,

R_f = resistance at any other temperature higher than the original temperature,

t = change of temperature in degrees centigrade,

a = temperature coefficient of resistance.

EXERCISE 16

Find the missing terms.

	R_f	R_i	a	t		
1.		200	.00406	25	Ans.	220.3.
2.		5000	.00406	85		6725.5.
3.	50.8	10	.0034			1200.
4.	60	55	.00406			22.39.
5.	220		.00406	25		199.7.
6.	25		.0034	650		7.78.

7. The resistance of a coil of copper wire at 39° is 300 ohms. What will be the resistance of the coil at 60° ($a = .003613$). Ans. 332.76.

8. The resistance of a coil of copper wire is 200 ohms at 40°. What will it be at 90° ($a = 0.003605$).

Ans. 236.05.

9. What will be the resistance of a copper wire at 25° if the resistance at 48° is 2.08 ohms? ($a = 0.003831$).

Ans. 1.91.

10. Solve the formula $R_f = R_t + R_t at$ for a and for t.

Ans. $a = \dfrac{R_f - R_t}{R_t t}$, $t = \dfrac{R_f - R_t}{R_t a}$.

43. Field Intensity Inside a Coil. The formula used for finding the field intensity inside a coil of wire is

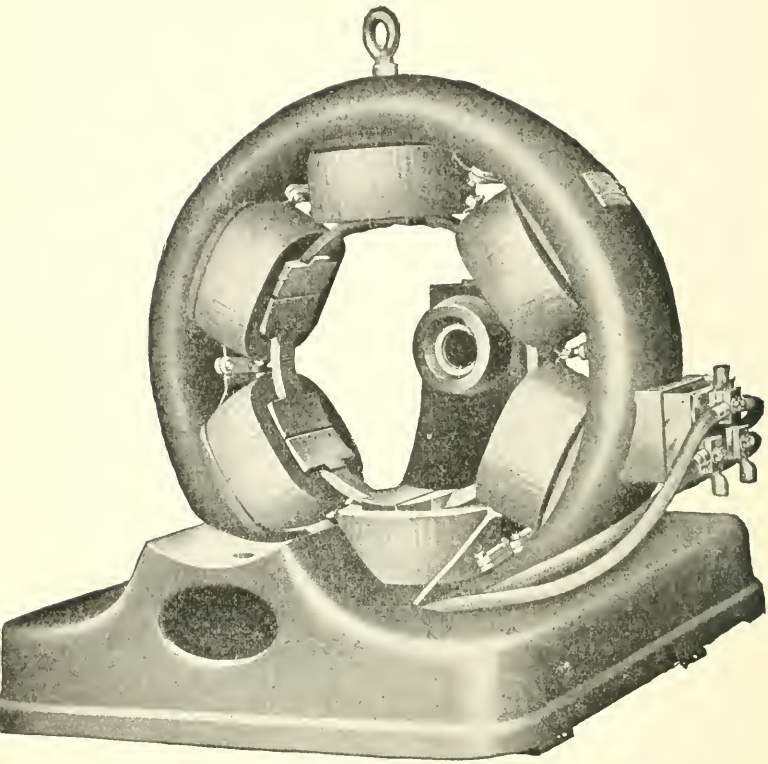
$$H = \frac{1.26NI}{L},$$

where, H = the field intensity inside the coil measured in gausses,
N = the number of turns of wire,
I = current through the coil in amperes,
L = length of the coil in centimeters.

EXERCISE 17

Find the missing terms:

	H	N	I	L	Ans.
1.		80	2	12	16.8
2.	40	115	1.5		5.434.
3.	33.35		1.75	1575	238.2.
4.	22.31	76		16.34	3.806.
5.		85	.85	20.1	4.53
6.	38.5	65.5	.741		1.59



Field Assembly of a Six-Pole Direct Current Generator with Slotted Poles.

7. Find the field intensity inside a coil of wire 12 cm. long having 100 turns and carrying a current of 0.5 ampere. Ans. 5.25.

8. A coil of wire is to have a field intensity of 40 gausses and is to be 15 cm. long, and carry a current of 1.2 amperes. Find the number of turns necessary. Ans. 396.8.

9. A coil 18 cm. long has 75 turns. What current will be necessary to give a field intensity of 650 gausses? Ans. 123.8.

44. Total Flux in a Magnetic Circuit. The following formulas are given for finding the total flux through a magnetic circuit:

$$\Phi = \frac{.4\pi NI}{R},$$

where, Φ = total flux through the magnetic circuit,
N = number of turns of wire in the magnetizing coil,
I = current in amperes in the magnetizing coil,
R = reluctance of magnetic circuit, computed as explained in the following equation.

In the above equation, the value of R, the reluctance, is found as follows:

$$R = \frac{l}{\mu A},$$

where, l = average length of magnetic circuit in centimeters.
μ = permeability of circuit.
A = average cross-sectional area of magnetic circuit.

When the circuit is made up of different parts, as the frame of the motor, air gaps, and core of the armature, Joint reluctance, or $R = R_1 + R_2 + R_3 + \ldots$

$$= \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \ldots$$

where R_1, R_2 and R_3 are reluctances of the different parts of the circuit and similarly l_1, l_2, etc., refer to the lengths in centimeters of different parts of the circuit.

EXERCISE 18

1. Find the total flux through a circuit composed of iron and an air gap, magnetized by 50 turns of wire carrying a current of 4 amperes, if the length of the iron is 36 cm., cross-section area 12 sq. cm., and $\mu = 2000$; and the length of the air gap is 2 cm., area 15 sq. cm., and $\mu = 1$.

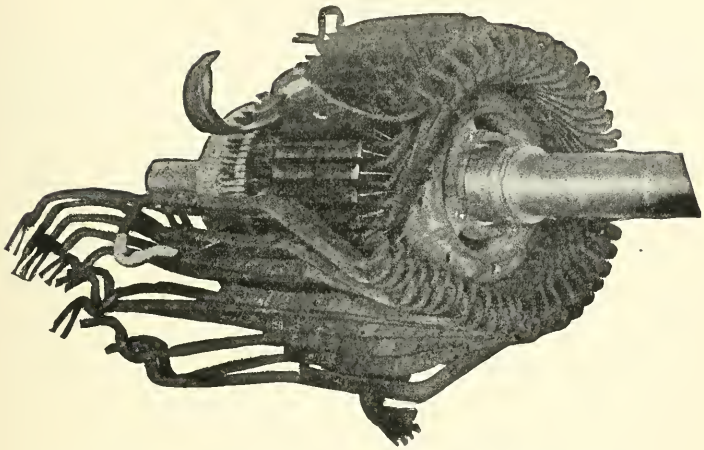
Ans. 1863.

2. Find the total flux in a motor if the path consists of the frame of the machine, the core of the coils, two air gaps between the coils and armature, and the armature. The magnetizing current is 24 amperes and has 200 turns. The cores of the machine have a cross section of 150 sq. cm., a length (including both cores) of 45 cm., and $\mu = 2000$. The frame of the machine has a length of 75 cm., an average area of 175 sq. cm., and $\mu = 1500$. The two air gaps have an area of 180 sq. cm., each has a length of 1.5 cm., and $\mu = 1$. The armature has an area of 160 sq. cm., a length of 40 cm., and $\mu = 2000$. Ans. 350,079.

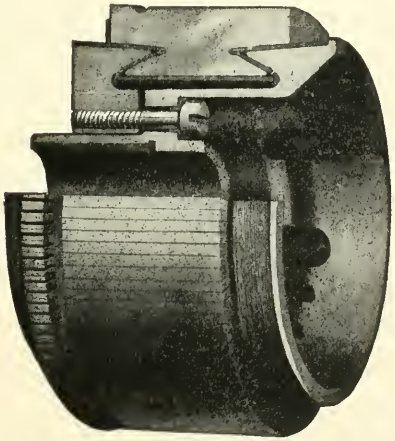
45. Armature Winding. The following formula is sometimes used in armature winding.

$$y = \frac{N \pm b}{2p} \pm 2m.$$

When the positive sign is used in the fraction, use the positive sign before $2m$, etc.



Partly Wound Armature showing Method of Assembling Coils.



Details of Commutator Construction.

EXERCISE 19

1. Evaluate the formula of Section 45, when $N = 120$, $b = 18$, $p = 3$, $m = 3$. Ans. 29 or 11.

2. Evaluate the formula of Section 45, when $N = 80$, $b = 24$, $p = 4$, $m = 2$. Ans. 17 or 3.

CHAPTER IV

POSITIVE AND NEGATIVE NUMBERS

46. Negative Numbers. The numbers used in arithmetic consist of a complete system of numbers and fractions, ranging in value from zero up. The order and value of these numbers may be represented along a straight line, as in Fig. 24.

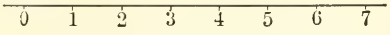


Fig. 24.

This system of numbers fails to express completely some numbers encountered in algebra. For example:

1. In the case of the thermometer there is a 5 above zero and a 5 below zero, represented on the scale as in Fig. 25.

The 5 above zero and the 5 below zero can be distinguished conveniently by +5 (called positive 5) and −5 (called negative 5). Then +12 means 12 above and −21 means 21 below zero.

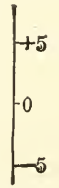


Fig. 25.

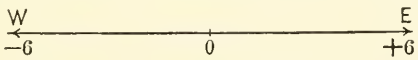


Fig. 26.

2. Two distances in opposite directions, as shown in Fig. 26, can be represented conveniently by positive and negative numbers, as +6 and −6.

Observe that

1. *The* (+) (*positive*) *and* (−) (*negative*) *signs have a new meaning, being used to distinguish things of opposite nature. The* (+) *and* (−) *will continue to be used as signs of addition and subtraction as well as signs of* **quality.**

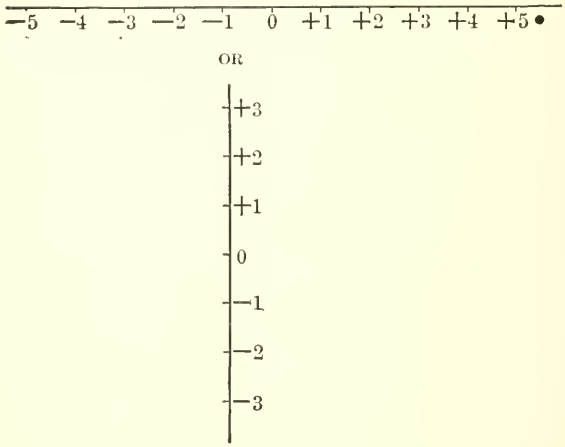


FIG. 27.

2. *The positive numbers are the same as the numbers used in arithmetic, and when no sign is expressed the* (+) *sign is understood.*

3. *The relative order and value of positive and negative numbers can be represented by the scale of Fig. 27.*

47. Addition. Algebraic addition is the combination of positive and negative numbers. For example:

1. A man travels 7 miles east and then 3 miles east from that point. This may be represented as in Fig. 28:

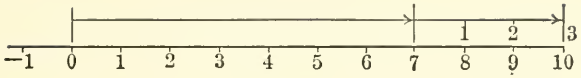


Fig. 28.

or

$$(+7)+(+3)=+10.$$

2. A man travels 7 miles west and then 3 miles west from that point. This may be represented as in Fig. 29:

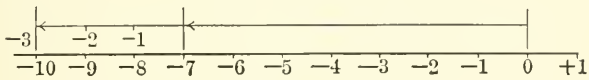


Fig. 29.

or

$$(-7)+(-3)=-10.$$

3. A man travels 7 miles east and then 3 miles west from that point. This may be represented as in Fig. 30:

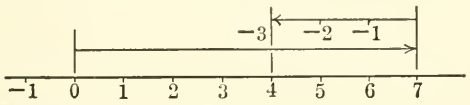


Fig. 30.

or

$$(+7)+(-3)=+4.$$

4. A man travels 3 miles east and then 7 miles west from that point. This may be represented as in Fig. 31:

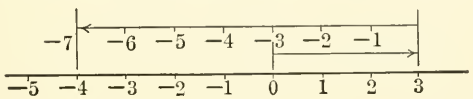


Fig. 31.

or

$$(+3)+(-7)=-4.$$

The problems above illustrate:

1. $(+7)+(+3)=+10$.
2. $(-7)+(-3)=-10$.
3. $(+7)+(-3)=+\ 4$.
4. $(+3)+(-7)=-\ 4$.

where the signs within the parentheses are signs of quality and the signs between the parentheses indicate algebraic addition. From these examples the following rule can be stated:

Rule. 1. *To add two numbers of like signs add the numbers as in arithmetic and give the result the common sign.*

2. *To add two numbers of opposite signs subtract the smaller from the larger and give to the result the sign of the larger.*

EXERCISE 1

Add

1. $+5,\ +12.$	Ans. $+17.$
2. $-5,\ -12.$	$-17.$
3. $+5,\ -12.$	$-7.$
4. $-5,\ +12.$	$+7.$
5. $-29x,\ 14x.$	$-15x.$
6. $-15d,\ 32d.$	$17d.$
7. $-3.71y,\ -5.23y.$	$-8.94y.$
8. $+9.21,\ -4.356.$	$4.854.$
9. $+22,\ -13,\ +24,\ -8.$	$+25.$

Note to Prob. 9. This problem can be solved most efficiently by adding all the positives and all the negatives and combining results.

10. $+34,\ -45,\ -17,\ +12.$	Ans. $-16.$
11. $5x,\ -7x,\ 14x.$	$12x.$
12. $-13x^2,\ -5x^2,\ 29x^2.$	$11x^2.$

13. $(a^3+3a^2b+3ab^2+b^3)+(2a^3+4a^2b-7ab^2)$
$+(-5a^3-ab^2+4b^3)$.

Solution. Arrange with similar terms in a column and add columns thus:

$$\begin{array}{r} a^3+3a^2b+3ab^2+\ b^3 \\ 2a^3+4a^2b-7ab^2 \\ -5a^3\qquad -\ ab^2+4b^3 \\ \hline -2a^3+7a^2b-5ab^2+5b^3 \end{array}$$

14. $(5x^2-3xy+2y^2)+(2x^2+3xy-y^2)+(x^2-4xy)$.
Ans. $8x^2-4xy+y^2$.

15. $(3a+5b-4c)+(2a+3c)+(a-12b)$. Ans. $6a-7b-c$.

16. $(5ax^2-3ay^2+2bz^2)+(7ay^2-6ax^2)$
$+(8bz^2+2ax^2-ay^2)$. Ans. $ax^2+3ay^2+10bz^2$.

48. Subtraction. Subtraction means to find the difference between two numbers, that is, the distance between the two numbers on the number scale. By reference to the number scale for positive and negative numbers find:

1. The distance to $+5$ from $+3$. Fig. 32.

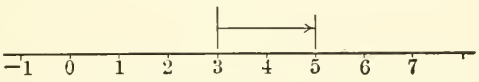


FIG. 32.

This is seen to be 2 in the positive direction. It may be computed by subtracting the second number from the first. Therefore

$$(+5)-(+3)=+2.$$

2. The distance to $+3$ from $+5$. Fig. 33.

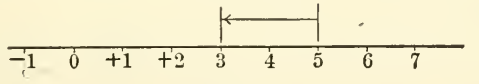


FIG. 33.

This is seen to be 2 in the negative direction. It is equivalent to subtracting the second number from the first. Therefore

$$(+3)-(+5)=-2.$$

3. The distance to -3 from $+5$. Fig. 34.

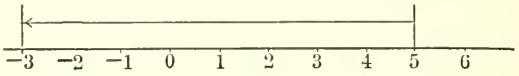


FIG. 34.

This is seen to be 8 in the negative direction. It is equivalent to subtracting the second number from the first. Therefore

$$(-3)-(+5)=-8.$$

4. The distance to $+5$ from -3. Fig. 35.

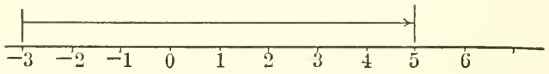


FIG. 35.

This is seen to be 8 in the positive direction. It is equivalent to subtracting the second number from the first. Therefore

$$(+5)-(-3)=+8.$$

EXERCISE 2

1. Find the following distances:

To	−5	+5	−10	+3	−8	+7
From	+5	−5	− 3	−8	+3	+4

2. Add

−5	+5	−10	+3	−8	+7
−5	+5	+ 3	+8	−3	−4
___	___	___	___	___	___

Compare the results in the two problems above and note the following rule:

Rule. *To subtract one number from another change the sign of the subtrahend and apply the rules for addition.*

EXERCISE 3

Subtract:

1. $+32$
   $+20$
   ——
   Ans. 12.

2. -24
   -18
   ——
   -6.

3. -42
   $+16$
   ——
   -58.

4. $+18$
   -22
   ——
   $+40$.

5. $+17x^2$
   $+5x^2$
   ——
   $12x^2$.

6. $-29ax$
   $+16ax$
   ——
   $-45ax$.

7. $+5ay^2$
   $-7ay^2$
   ——
   $12ay^2$.

8. $+2a-3b$
   $+a+b$
   ——
   $a-4b$.

9. $(-5)-(-7)$. 2.
10. $(+8)-(-3)$. 11.
11. $(-12)-(+16)$. -28.
12. $(3x+7y)-(2x+3y)$. $x+4y$.

13. $(3x^2-4xy+7y^2)-(2x^2+3xy-12y^2)$.
Ans. $x^2-7xy+19y^2$.

14. $(5a^2+6ab+2b^2)-(3a^2-2ab+b^2)-(2a^2-8ab+3b^2)$.
Ans. $6a^2+12ab$.

15. $(12x^3-7x^2+4x)-(2x^3+3x^2-8x)+(4x^3-5x^2+6x)$.
Ans. $14x^3-15x^2+18x$.

16. $(a+b)-(c-d)$. Ans. $a+b-c+d$.

NOTE. Subtraction will remove the parentheses but there are no similar terms that can be combined.

17. $x^2+3x-(-5+y)$. Ans. $x^2+3x+5-y$.

18. $4+3x-(2x-5)$. $9+x$.

Solve and check:

19. $10x-(3x-4)=(4x+4)+7$. $2\frac{1}{3}$.

NOTE. First remove the parentheses by changing the signs of the subtrahend thus:

$$10x-3x+4=4x+4+7.$$

20. $6-(3n-4)=(2n-3)-(n-1)$. Ans. 3.

21. $(2x+3)-(-3x-2)=25$. 4.

22. $(5x-3)-7=(2x+5)$. 5.

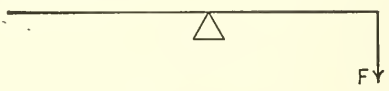


FIG. 36.

49. Multiplication. When a force is applied to a lever, it will cause the lever to turn or to have a tendency to turn. If the force is doubled, the tendency to turn will be doubled; or if the length of the lever is doubled, the tendency to turn will be doubled. The tendency of a lever to turn is called the **turning moment** or **leverage.** The length of the lever from the **fulcrum** (the turning point) to the force is called the **lever arm.** See Fig. 36.

The leverage caused by a force is equal to the force times the arm.

An arm to the right of the fulcrum is considered positive and an arm to the left of the fulcrum is considered negative. An upward pulling force is considered positive and a downward pulling force is considered negative. These assumptions are the same as the numbers on the number scale.

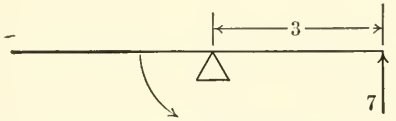


Fig. 37.—Positive Leverage.

A positive force on a positive arm must give a positive leverage since $(+4)(+7)=+21$ (Arithmetic). See Fig. 37.

Note. A leverage in the direction opposite to the motion of the hands of a clock (counter clockwise direction) is a positive leverage. Then a leverage in the clockwise direction must be considered negative.

A negative force on a negative arm will cause the lever to turn in the same direction (counter clockwise) giving likewise a positive leverage, Fig. 38. Therefore

$$(-3)(-7)=+21.$$

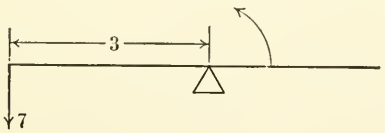


Fig. 38.—Positive Leverage.

A negative force on a positive arm will cause the lever

to turn in the opposite direction (clockwise) giving a negative leverage, Fig. 39. Therefore

$$(+3)(-7) = -21.$$

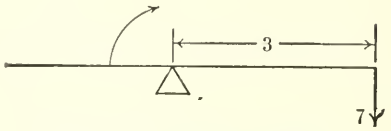


Fig. 39.—Negative Leverage.

A positive force on a negative arm will cause the lever to turn in the clockwise direction, Fig. 40. Therefore

$$(-3)(+7) = -21.$$

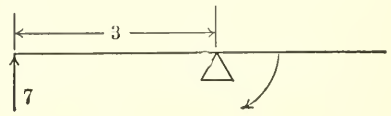


Fig. 40.—Negative Leverage.

The four preceding examples show that

1. $(+7)(+3) = +21.$
2. $(-7)(-3) = +21.$
3. $(-7)(+3) = -21.$
4. $(+7)(-3) = -21.$

From these problems a law of signs for multiplication can be stated.

50. Law of Signs for Multiplication.

1. *If two factors have like signs their product is positive.*
2. *If two factors have unlike signs their product is negative.*

EXERCISE 4

Multiply:

1.	$(3)(+12)$.	Ans. $+36$.
2.	$(-3)(-12)$.	$+36$.
3.	$(-\frac{2}{3})(+\frac{3}{4})$.	$-\frac{1}{2}$.
4.	$(2.5)(-4)$.	-10.
5.	$(-5.6)(-2.3)$.	12.88.
6.	$(3\frac{2}{3})(-5)$.	$18\frac{1}{3}$

51. Law of Exponents. By definition of an exponent

x^3 means xxx

and

x^4 means $xxxx$.

Therefore

$$(x^3)(x^4) = (xxx)(xxxx) = x^7.$$

From this and similar problems a law of exponents can be stated:

Rule. *To multiply powers of the same base add their exponents.*

EXERCISE 5

Multiply:

1.	x^7x^4.	Ans. x^{11}.
3.	$(a^3)(-a^2)$.	$-a^5$.
3.	$(-y^2)(-y)$.	$+y^3$.
4.	x^3x^5x.	x^9.

52. Multiplication of Monomials. When different bases occur in the factors, their product can be indicated only. Thus

$$(x^3)(y^2) = x^3y^2.$$

When several different bases occur in each factor, as

$$(3a^2b^3r)(4a^3b^4x^6),$$

only the powers of the same base can be combined. For example,

$$(3a^2b^3r)(4a^3b^4x^6) = 3 \times 4a^2a^3b^3b^4rx^6 = 12a^5b^7rx^6.$$

NOTE. Expressions whose parts are not separated by the (+) or (−) signs are called **monomials.**

From the above example a rule for the multiplication of monomials can be stated.

Rule. *To multiply two monomials multiply their numerical coefficients and annex all the different bases, giving to each base an exponent equal to the sums of the exponents of that base in the two factors.*

EXERCISE 6

Multiply:

1. $(4x^3)(5x^4)$.	Ans. $20x^7$.
2. $(12x^2y^3)(2x^3y^5)$	$24x^5y^8$.
3. $(5a^3x^3y^4)(6a^2xz^2)$.	$30a^5x^4y^4z^2$.
4. $(-2ab^2)(13b^2c)$.	$-26ab^4c$.
5. $(5xy)(-4xz)$.	$-20x^2yz$.

53. Law of Leverages. If two forces act upon the same lever at the same time, the lever will be in balance when the

positive leverages equal the negative leverages; that is, when the sum of all the leverages equals zero. Thus in Fig. 41:

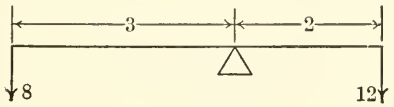


FIG. 41

The leverage caused by the force $8 = (-3)\ (-8) = +24$.

The leverage caused by the force $12 = (-12)(+2) = -24$.

$$(+24)+(-24)=0,$$
$$0=0.$$

Therefore the lever will balance.

EXERCISE 7

1. Find the sum of all the leverages in Fig. 42.

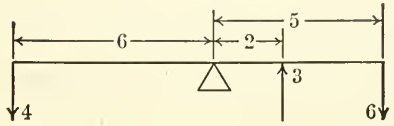
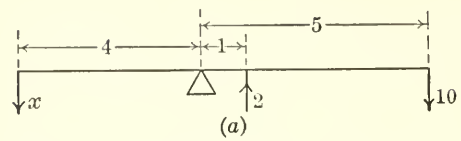


FIG. 42.

Will the lever balance? Why?

2. Find the value of x that will make the levers balance in Figs. 43 (*a*)–43 (*e*).



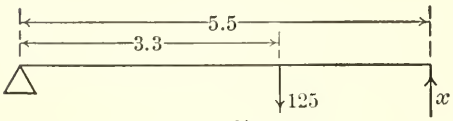
(a)

Solution.

$4x+2-50=0$

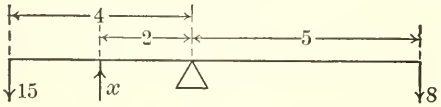
$4x=48$

$x=12.$



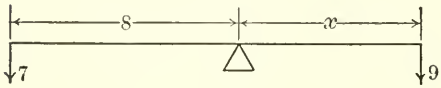
(b)

Ans. 75.



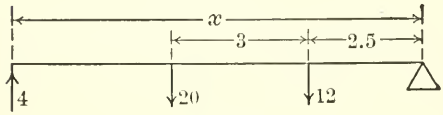
(c)

Ans. 10.



(d)

Ans. $6\frac{2}{9}$.



(e)

Ans. 35.

FIG. 43.

54. Multiplication of Polynomials by Monomials. The perimeter of a rectangle is $2(a+b)$ and also $2a+2b$. Fig. 44.

Therefore

$$2(a+b)=2a+2b.$$

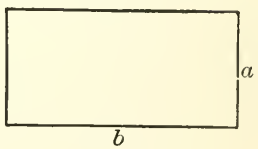


FIG. 44.

Expressions like $(a+b)$ and (x^2-3x+4) which consist of two or more parts added or subtracted, are called *polynomials.*

From the above problem a rule for multiplication of a polynomial by a monomial can be stated:

Rule. *To multiply a polynomial by a monomial multiply each term of the polynomial by the monomial.*

EXERCISE 8

Multiply:

1. $a^2+3ab+4b^2$ by 5. Ans. $5a^2+15ab+20b^2$.
2. $x^2-3xy+4y^2$ by $5x$. $5x^3-15x^2y+20xy^2$.
3. $a^3-a^2b-2ab^2$ by $-2ab$. $-2a^4b+2a^3b^2+4a^2b^3$.
4. $x^2-7x^2y^2+4y^2$ by $3x^2y^3$. $3x^4y^3-21x^4y^5+12x^2y^5$.

55. Multiplication of a Polynomial by a Polynomial. The product $(y+6)(x+2)$ can be represented by the rectangle, Fig. 45, whose length is $y+6$ and whose width is $x+2$. The area of the rectangle is the product $(y+6)(x+2)$. The area is also the sum of the areas of the four small rectangles, that is, $xy+6x+2y+12$. Therefore

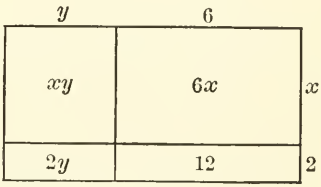


FIG. 45.

$$(y+6)(x+2)=xy+6x+2y+12$$

From the above problem the following rule can be stated:

Rule. *To multiply a polynomial by a polynomial multiply each term of the one by every term of the other and combine the results.*

The work can be arranged conveniently thus:

$$\begin{array}{ll} x-2 & \\ x+3 & \\ \hline x^2-2x & \text{(multiplying } x-2 \text{ by } x\text{)} \\ \quad 3x-6 & \text{(multiplying } x-2 \text{ by } 3\text{)} \\ \hline x^2+x-6 & \text{(combining similar terms)} \end{array}$$

EXERCISE 9

Multiply:

1. $(x+3)(x+4)$.	Ans. $x^2+7x+12$.
2. $(x-5)(x-2)$.	$x^2-7x+10$.
3. $(x-y)(x-y)$.	$x^2-2xy+y^2$.
4. $(2a+3b)(3a-4b)$.	$6a^2+ab-12b^2$.
5. $(a-b)(a-b)$.	$a^2-2ab+b^2$.
6. $(x+2)(x+2)$.	x^2+4x+4.
7. $(y-3(y-3)$.	y^2-6y+9.
8. $(n-5)(n-5)$.	$n^2-10n+25$.

56. Division. The division of positive and negative numbers requires a law of signs and a law of exponents which are developed by a study of the laws of signs and exponents for multiplication. Division is the opposite of multiplication, hence,

Since $(+5)(+6)=+30$ it follows $(+30)\div(+6)=+5$.
Since $(+5)(-6)=-30$ it follows $(-30)\div(-6)=+5$.
Also $(-30)\div(+5)=-6$.
Since $(-5)(-6)=(+30)$ it follows $(+30)\div(-6)=-5$.

From these problems a law of signs for division can be stated:

57. Law of Signs for Division.

1. *If two numbers have like signs their quotient is positive.*

2. *If two numbers have unlike signs their quotient is negative.*

EXERCISE 10

Divide:

1. $(+25)\div(+5)$.	Ans.	$+5$.
2. $(-63)\div(-3)$.		$+21$.
3. $(-24)\div(+5)$.		-4.8.
4. $(+38)\div(-4)$.		-9.5.
5. $(-36)\div(-4)$.		$+9$.
6. $(-24)\div(-3)$.		8.

58. Exponents in Division. By the law of exponents for multiplication,

$$x^7x^4 = x^{11}$$

Therefore

$$x^{11}\div x^7 = x^4$$

and

$$x^{11}\div x^4 = x^7$$

From this problem the following law of exponents for division can be stated:

1. *To divide powers of the same base subtract the exponent of the divisor from the exponent of the dividend.*

2. *The quotient of powers of different bases can be indicated only.*

59. Division of Monomials.

Example. Divide $24a^3x^2y^7z^4$ by $8a^2bx^2y^5xz^6$.

Solution.

$$\frac{\cancel{3}\ \cancel{a}\qquad \cancel{y^2}}{\cancel{8}\ \cancel{a^2}\,b\ \cancel{x^2}\ \cancel{y^5}\ \cancel{z^6}\ \cancel{z^2}} = \frac{3ay^2}{bz^2}$$

Rule. *To divide a monomial by a monomial write the two monomials as a fraction and cancel equal factors.*

EXERCISE 11

Divide:

1. $12a^3x$ by $3a$.	Ans. $4a^2x$.
2. $35a^2x^7y^8$ by $-7a^2x^3y^6$.	$-5x^4y^2$.
3. $-46a^3b^5m^3$ by $4a^3bm$.	$-11.5b^4m^2$.
4. $-75x^7b^4c^3$ by $-10xb^4$.	$7.5x^6c^3$.
5. $38r^2s^3c$ by $-19r^2s^3c$.	-2.
6. $27x^3$ by $9y^3$.	$3\frac{x^3}{y^3}$.

60. Division of a Polynomial by a Monomial. By the rules for multiplication

$$5(x-2y) = 5x-10y$$

Therefore:

$$(5x-10y) \div 5 = x-2y$$

Rule. *To divide a polynomial by a monomial divide each term of the polynomial by the monomial.*

The work can be arranged conveniently thus:

$$\frac{10x^2y^5-15x^3y^4+25x^5y^2}{5xy^2} = 2xy^3-3x^2y^2+5x^4$$

EXERCISE 12

Divide:

1. $\dfrac{x^3-2x^2+x}{x}$. Ans. x^2-2x+1.

2. $\dfrac{ay^2-3ay+4a}{a}$. y^2-3y+4.

3. $\dfrac{x-xy}{x}$. $1-y$.

4. $\dfrac{R_t+R_tat}{R_t}$. $1+at$.

5. $\dfrac{7a^3x^2-14a^2x^3+28a^5}{-7a^2}$. $-ax^2+2x^3-4a^3$.

6. $\dfrac{6x^2y^2z^2-12xy^2z^3-15x^3y^2z}{3xyz}$. $2xyz-4yz^2-5x^2y$.

CHAPTER V

RATIO AND PROPORTION

61. Ratio. The relation of one number to another number of the *same kind* is called a *ratio.* A ratio is expressed as 5 : 3, read the ratio of 5 to 3, or as $\frac{5}{3}$. The value of a ratio is found by writing it as a fraction and reducing the fraction to its lowest terms.

EXERCISE 1

1. Find the value of the ratio $\frac{2}{3}\frac{4}{2}$.

Solution. $\frac{2}{3}\frac{4}{2}=\frac{3}{4}$

2. Find the value of the ratio $\dfrac{7\times 9}{8\times 7}$

Solution. $\dfrac{7\times 9}{8\times 7}=\dfrac{9}{8}=1\frac{1}{8}$

3. Find the ratio of the areas of two rectangles having dimensions 15 by 18 and 24 by 12.

Solution. $\dfrac{\overset{5}{\cancel{15}}\times\overset{3}{\cancel{18}}}{\underset{8}{\cancel{24}}\times\underset{4}{\cancel{12}}}=\dfrac{15}{16}$

4. Find the ratio of the circumferences of two circles having diameters of 3″ and 8″. Ans. $\frac{3}{8}$.

5. Find the ratio of the areas of two circles having radii of 7″ and 12″. Ans. $\frac{49}{144}$.

6. Find the ratio of the two values of W in the formula

$$W = .26bh,$$

when $b = 18$, $h = 25$, and when $b = 15$, $h = 42$. Ans. $\frac{5}{7}$.

62. Percentage. It is often more significant to know the relative number than to know the absolute number. For example, a city may increase in population 25,000 in ten years, but it is more significant of the growth of a city to say that it increased in population 60 per cent or 10 per cent. When a chemist makes an analysis he records his results as 18 per cent iron in the ore he analyzes, rather than 25 grams of iron in the sample he analyzed. The relative number is called *per cent* and is found by taking the ratio of a part to the whole. For example, if 25 grams of iron are found in 175 grams of ore, then the ratio which gives the per cent of iron in the ore is

$$\frac{\text{grams of iron}}{\text{grams of ore}} = \frac{25}{175} = \frac{1}{7} = .142+ = 14+ \text{ per cent.}$$

Observe that:

1. $\frac{25}{175}$, $\frac{1}{7}$, .142+, and 14 per cent are different ways of expressing the same ratio.

2. Per cent is expressed on the basis of hundredths, the ratio .14 means 14 per cent, the ratio .855 means $85\frac{1}{2}$ per cent.

EXERCISE 2

1. If $62\frac{1}{2}$ tons of iron are obtained from 625 tons of ore, what is the per cent of iron in the ore? Ans. 10.

2. If 3 gallons of water are added to 22 gallons of alcohol, what is the per cent of alcohol in the mixture? Ans. 12.

3. Potassium chloride is composed of 39 parts of potassium to 35.5 parts chlorine. Find the percentage of chlorine in potassium chloride. Ans. 47.

4. Babbitt metal is composed of 92 parts of tin to 8 parts of copper and 4 parts of antimony. Find the percentage of copper in babbitt metal. Ans. 8.

5. If 5 lbs. of vegetables lose 5 oz. in drying, what part of the original weight was water? Ans. 6 per cent.

6. An ore analyzed 23 per cent iron. How many lbs. of iron could be obtained from 1 ton of ore?

Solution. If x lbs. could be obtained,

Then $$\frac{x}{2000} = .23$$

$$x = 460$$

7. If water is $87\frac{1}{2}$ per cent oxygen, how many grams of oxygen could be obtained from 17 grams of water?

Ans. $14\frac{7}{8}$.

8. If potassium nitrate is 39 per cent potassium, how many grams of potassium nitrate would be required to give 24 grams of potassium? Ans. 61.5.

9. If ore contains 14 per cent iron, how many tons of ore are required to get 125 tons of iron? Ans. 892.86.

10. If 2 tons of coal give 625 lbs. of ash, what per cent of the coal is ash? Ans. 16.

63. Efficiency. The efficiency of a machine is the ratio of the work done by the machine to the work put into the machine. Efficiency is expressed as per cent. The rule in the formula form is,

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

EXERCISE 3

Find the missing values in the following:

	LOSSES	INPUT	EFFICIENCY	
1.	950	6850		Ans. 86%.
2.		6950	75	1737.
3.	692	6820		89%.
4.	800		90	8000.

5. The losses of a motor while running are 500 watts, the input is 4500 watts, what is the efficiency of the machine? Ans. 89 per cent.

6. The efficiency of a motor is 79 per cent. The input is 60,000 watts, what is the output? Ans. 47,400.

7. A certain acid mixture requires 2 parts of acid to 3 parts of water. Find what per cent of the mixture is acid and what per cent is water. Ans. 40, 60.

8. In the above problem find how much acid and how much water are in 14 gallons of the mixture. Ans. 5.6, 8.4.

64. Separating in a Given Ratio. Problem 8 above could be solved more readily as follows:

Let $2x$ = amount of acid

Then
$$3x = \text{amount of water}$$
$$2x + 3x = 14 \text{ gallons}$$
$$5x = 14$$
$$x = 2\tfrac{4}{5}$$
$$2x = 5\tfrac{3}{5} \text{ gallons of acid}$$
$$3x = 8\tfrac{2}{5} \text{ gallons of water}$$

EXERCISE 4

1. Divide 35 into two parts in the ratio of 2 to 3. Ans. 14, 21.

2. Divide 180 in the ratio 4 to 5. Ans. 80, 100.

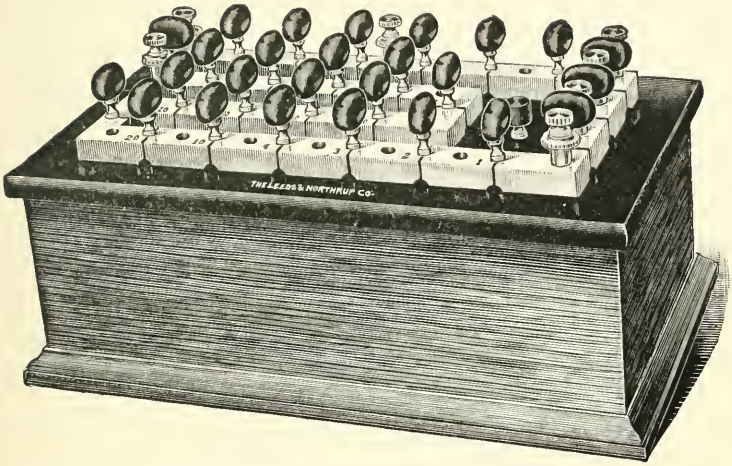
3. Bronze is composed of 11 parts of tin to 39 parts of copper. Find the number of lbs. of tin and copper in 625 lbs. of bronze. Ans. 137.5, 487.5.

4. Eighteen carat gold is composed of 18 parts of pure gold to 6 parts of other metal. How much pure gold is contained in 4.8 oz. of the alloy? Ans. 3.6.

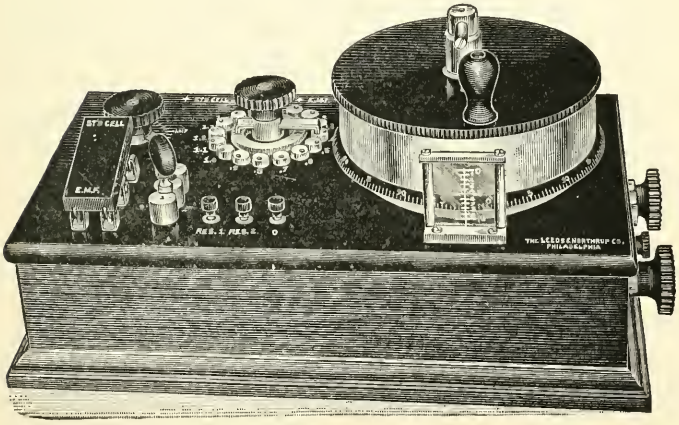
5. In an electric circuit the fall of potential over any two of the parts is in the ratio of the resistances. 24 ohms and 40 ohms are joined in a circuit. The drop of the potential over both resistances is 90 ohms, find the drop over each resistance. Ans. $33\frac{3}{4}$, $56\frac{1}{4}$.

6. In a divided circuit the current divides in the ratio of the resistances, the larger current taking the path of the smaller resistance. If the two resistances of a divided circuit are 24 ohms and 36 ohms, and the total current flowing is 8 amperes, find how much current will flow in each branch. Ans. 4.8, 3.2.

65. Proportion. In two samples of the same kind of bronze the ratios of the copper to the tin are equal. When two ratios are equal they form a *proportion*. A proportion



Wheatstone Bridge.



Leeds and Northrup Potentiometer.

is written as $\frac{3}{7}=\frac{9}{21}$, and read the ratio of 3 to 7 equals the ratio of 9 to 21; or 3 : 7 :: 9 : 21, and read 3 is to 7 as 9 is to 21. The first and last terms of a proportion are called **extremes** and the second and third are called **means.**

A proportion is used in finding ratios of quantities when some other ratio between the same two quantities is known.

For example: In bronze 11 parts of tin combine with 39 parts of copper. The same ratio holds for any quantity of bronze. How many parts of tin will combine with 260 lbs. of copper?

Solution. Let x = the number of pounds of tin.

Then the ratio $\frac{11}{39}$ = the ratio $\frac{x}{260}$

$$\frac{11}{39}=\frac{x}{260}$$

$$2860=39x$$

$$x=73.33+ \text{ lbs. of tin}$$

Observe that:

1. *A proportion is an equation.*

2. *A proportion can be cleared of fractions by multiplying both members by the product of the denominators.*

3. *To clear a proportion of fractions write the product of the means equals the product of the extremes.*

66. The Wheatstone Bridge. In measuring resistance the unknown resistance is placed in a Wheatstone bridge, as shown in Fig. 46, where any three of the resistances are known and the fourth resistance is the resistance to be

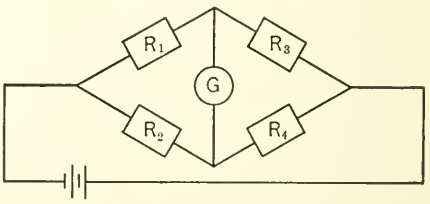


Fig. 46.—Wheatstone Bridge.

measured. The three known resistances are adjusted to make $\frac{R_1}{R_2}=\frac{R_3}{R_4}$. The galvanometer G indicates when the resistances are so adjusted that the ratios are equal. The proportion $\frac{R_1}{R_2}=\frac{R_3}{R_4}$ is used as a formula.

EXERCISE 5

Find the missing numbers in the following:

	R_1	R_2	R_3	R_4	Ans.
1.		5	4	3	6.67
2.	1.5		3.5	2.5	1.07
3.	9.85	6.8	9.9		6.84
4.	1.5	2.35		5	3.19
5.	8.7		9.5	45	41.5
6.	160	32	43		8.6
7.	650	80	160		19.7
8.	60		235	25	6.38
9.	4.75		6.843	5	3.47
10.		275	6.625	9.375	195.1

11. If $\frac{R_1}{R_2}=100$ and $R_4=4.26$, find R_3. Ans. 426.

67. Reduction to Fractions of a Given Denominator. It is often necessary to express a fraction or a decimal in halves, fourths, sixteenths, etc.

Example. How many eighths in $\frac{4}{5}$ of an inch?

Solution. If there are x eighths, then

$$\frac{x}{8}=\frac{4}{5}$$

$$5x=32$$

$$x=6.4$$

EXERCISE 6

1. How many 64ths in $\frac{3}{5}$ of an inch? Ans. 38.4.
2. How many 32ds in $\frac{72}{1000}$ of an inch? 23.
3. How many 64ths in .365 of an inch? 23.36.
4. How many 16ths in .82 of an inch? 13.12.
5. How many 64ths in $\frac{7}{12}$ of an inch? 37.3.
6. How many thousandths in $\frac{1}{64}$ of an inch? 15.625.
7. How many thousandths in $\frac{3}{16}$ of an inch? 187.5.

68. Writing a Proportion. In writing a proportion two types of problems will be encountered. First, when the two quantities are so related that an increase or a decrease in one will produce the same kind of a change on the other.

For example: If 20 men assemble 8 machines in a day, more men could assemble more machines, and less men less machines. This is called a **direct proportion.**

Secondly, when two quantities are so related that an increase or a decrease in the one will produce the opposite change in the other.

For example: It takes 30 men 12 days to assemble a machine, more men could do it in less time and for less men it would take more time. This is called an **inverse proportion.**

Example 1. 12 feet of angle iron weighs 44 lbs., how much will 30 ft. weigh?

Solution. $$\frac{12 \text{ ft}}{30 \text{ ft}} = \frac{44 \text{ lbs.}}{x \text{ lbs.}}$$

Solve the equation for x. Ans. 110.

Every ratio *must be a comparison of similar things,* and

in every proportion both ratios *must be written in the same order of value*, that is:

$$\frac{\text{Small length}}{\text{Large length}} = \frac{\text{Small weight}}{\text{Large weight}}$$

or

$$\frac{\text{Large length}}{\text{Small length}} = \frac{\text{Large weight}}{\text{Small weight}}$$

Example 2. If 30 men do a piece of work in 12 days, how long will it take 20 men to do the same work?

Solution.

$$\frac{\text{Large number of men}}{\text{Small number of men}} = \frac{\text{Large number of days}}{\text{Small number of days}}$$

That is

$$\frac{30}{20} = \frac{x}{12}$$

Solve for x. Ans. 18.

Observe that:

1. In the direct proportion the 44 lbs. corresponds to 12 ft. and x lbs. to 30 ft. and the corresponding numbers are arranged directly across from each other thus:

$$\frac{12 \longleftrightarrow 44}{30 \longleftrightarrow x}$$

2. In the inverse proportion the 30 men correspond to 12 days and 20 men to x days and the corresponding numbers are arranged diagonally thus:

$$\frac{30 \searrow \quad \nearrow x}{20 \nearrow \quad \searrow 12}$$

EXERCISE 7

Solve by proportion:

1. If a steel rail 3 ft. long weighs 112 lbs., how much will a rail 20 ft. long weigh? Ans. 746.7.

2. 20 men do a piece of work in 54 days. How many men would it take to do it in 30 days? Ans. 36.

3. The volume of a quantity of gas is inversely proportional to the pressure upon it. If a quantity of gas measures 350 cu. ft. at 15 lbs. pressure, how many cubic feet will it measure at 25 lbs. pressure? Ans. 210.

4. The pressure of 627 cu. ft. of gas is to be reduced from 35 lbs. to 24 lbs., what will be the volume.

Ans. 1164.37.

5. The volume of a gas is directly proportional to the absolute temperature when the pressure is constant. If a quantity of gas occupies 125 cu. ft. at 278°, what will be its volume at 316°? Ans. 124.09.

6. If a quantity of gas occupies 28 cu. ft. at a temperature of 250°, what will be the temperature when it occupies 30 cu. ft.? Ans. 267.85.

7. If the resistance of 25 ft. of wire is 11.2 ohms, what will be the resistance of 83 ft. of the same wire?

Ans. 37.184.

8. An investment produces an income of \$600 at $3\frac{1}{2}$ per cent. What would it produce at 5 per cent?

Ans. 857.14.

CHAPTER VI

CUTTING SPEED, PULLEYS AND GEARS

69. Rim Speed. When work is turned in a lathe, the work must pass by the point of the cutting tool at a speed which will complete the work in the shortest possible time without injury to the work or the tools used. When the work is turned one complete revolution, a point on the surface of the work travels a distance equal to the circumference of the work. In one minute the point would travel a distance equal to the circumference of the work times the number of revolutions per minute (R.P.M.). The same applies to emery wheels, grindstones, and pulleys. The distance traveled in one minute by a point on the circumference of any revolving object is called **rim speed, surface speed** or **cutting speed.** Rim speed must be expressed in feet per minute.

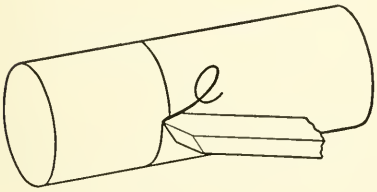


FIG. 47.

Rule. *To find rim speed multiply the circumference of the revolving object by the number of revolutions per minute.*

By formula:

$$RS = C \cdot (\text{R.P.M.})$$

where C = circumference in feet, and RS = rim speed. Or

$$RS = \frac{C \cdot (\text{R.P.M.})}{12}$$

where C = circumference in inches.

But since $C = \pi d$, where d is the diameter,

$$RS = \frac{\pi d(\text{R.P.M.})}{12}$$

Example 1. The diameter of a bolt being turned in a lathe is 3 ins. If it makes 160 R.P.M., what is the rim speed?

Solution.

$$RS = \frac{\pi d(\text{R.P.M.})}{12}$$

$$RS = \frac{3.1416 \cdot 3 \cdot 160}{12} \qquad \text{(Substitution.)}$$

$$RS = 125.664 \text{ ft. per min.}$$

Example 2. At how many revolutions per minute must a 4-in. bolt be turned to give a cutting speed of 60 ft. per min.?

Solution.

$$RS = \frac{\pi d(\text{R.P.M.})}{12}$$

$$60 = \frac{3.1416 \cdot 4 \cdot (\text{R.P.M.})}{12} \qquad \text{(Substitution.)}$$

$$60 \cdot 12 = 3.1416 \cdot 4 \cdot (\text{R.P.M.}) \qquad \text{(Clearing of fractions.)}$$

$$\frac{60 \cdot 12}{3.1416 \cdot 4} = \text{R.P.M.} \qquad \text{(Dividing by the coefficient of R.P.M.)}$$

$$\text{R.P.M.} = 57.6 + \text{in.}$$

Example 3. A pulley on a shaft turning at 140 R.P.M. is to furnish a rim speed of 1500 ft. per min. Find the diameter of the pulley that must be placed on the shaft.

Solution.

$$RS = \frac{\pi d(\text{R.P.M.})}{12}$$

$$15.00 = \frac{3.1416 \cdot d \cdot 140}{12} \qquad \text{(Substitution.)}$$

$$1500 \cdot 12 = 3.1416 \cdot d \cdot 140 \qquad \text{(Clearing of fractions.)}$$

$$\frac{1500 \cdot 12}{3.1416 \cdot 140} = d \qquad \text{(Dividing by the coefficient of } d.\text{)}$$

$$d = 40.9+ \text{ inches}$$

NOTE. In rough work where the exact answer is not required the following formula is used:

$$R.S. = \frac{d(\text{R.P.M.})}{4}, \quad \text{since} \quad \frac{3\ 1416}{12} \text{ is approximately equal to } \tfrac{1}{4}.$$

EXERCISE 1

Find the exact results in the following:

1. Find the cutting speed of a 5-in. cylinder being turned at 75 R.P.M. Ans. 98.175 ft. per min.

2. The armature of a dynamo is 16 ins. in diameter and runs at 1350 R.P.M. Find the rim speed. Ans. 5654.9 ft. per min.

3. At how many R.P.M. should a 6-in. cylinder be turned to give a cutting speed of 60 ft. per minute? Ans. 38.2.

4. An 8-in. emery wheel has a rim speed of 4000 ft. per minute. How many R.P.M. does it make. Ans. 1909.

5. A shaft is running at 175 R.P.M. How large a pulley must be placed on this shaft to give a rim speed of 2250 ft. per minute? Ans. 49.11 ins.

6. A 30-in. pulley runs at 250 R.P.M. Find the rim speed. Ans. 1963.5.

7. The pulley of Problem 6 is belted to a 15-in. pulley. What is the speed of the belt? What is the rim speed of the 15-in. pulley? Ans. 1963.5; 1963.5.

8. Find the R.P.M. of the 15-in. pulley. Ans. 500.

70. Pulley Speeds. Problems 6, 7 and 8 above illustrate the fact that if a 30-in. pulley is belted to a 15-in. pulley the R.P.M. of the 15-in. pulley will be twice the R.P.M. of the 30-in. pulley. Or, in general:

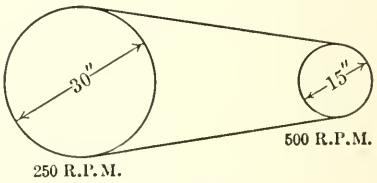


FIG. 48.

Rule. *When two pulleys are belted together the R.P.M. vary inversely as the size of the pulleys.*

Example 1. A 20-in. pulley running at 180 R.P.M. drives an 8-in. pulley. Find the R.P.M. of the 8-in. pulley.

Solution. Let x = the R.P.M. of the 8-in. pulley. Then since 180 = R.P.M. of the 20-in pulley,

$$\frac{x}{180} = \frac{20}{8}$$

$$8x = 3600$$

$$x = 450 \text{ R.P.M.}$$

EXERCISE 2

1. A 32-in. pulley running at 150 R.P.M. drives a 22-in. pulley. Find the R.P.M. of the 22-in. pulley. Ans. 218+.

2. A 30-in. pulley is to drive a 12-in. pulley at 800 R.P.M. Find the R.P.M. of the 30-in. pulley. Ans. 320.

3. A 22-in. pulley on a shaft running at 264 R.P.M. is to drive a machine at 570 R.P.M. Find the size of the pulley on the machine. Ans. 10.17.

4. A line shaft running at 150 R.P.M. is to drive a machine having a 14-in. pulley at 375 R.P.M. Find the size pulley that will be required on the shaft. Ans. 35 ins.

5. An electric motor running at 1250 R.P.M. and having a 16-in. pulley is to drive a line shaft at 175 R.P.M. Find the size of the pulley on the line shaft. Ans. 114.3 ins.

6. A 30-in. pulley running at 240 R.P.M. drives a 24-in. pulley. Find the R.P.M. of the 24-in. pulley. Ans. 300.

71. Gears. In machines where driving is done by means of gears it will be seen that, if two gears are meshed together, the smaller gear will have the greater R.P.M. Sizes of gears are measured by the number of teeth.

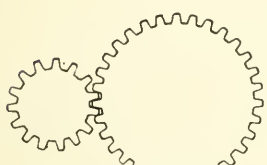


FIG. 49.—Gears in Mesh.

Rule. *When two gears run together, the R.P.M. varies inversely as the number of teeth.*

EXERCISE 3

1. A 48-tooth gear is driving a 72-tooth gear. Find the R.P.M. of the 72-tooth gear if the 48-tooth gear is running 160 R.P.M. Ans. 106.7.

2. A 72-tooth gear running at 190 R.P.M. is to drive another gear at 360 R.P.M. Find the number of teeth in the second gear. Ans. 38.

3. A 26-tooth gear running at 105 R.P.M. is to drive a 14-tooth gear. Find the R.P.M. of the 14-tooth gear. Ans. 195.

EXERCISE 4

(Miscellaneous Problems)

1. A locomotive has a 6-foot drive wheel. Find the R.P.M. of the wheel when the engine is running at 50 miles per hour. Ans. 700.2.

2. Cone pulleys:

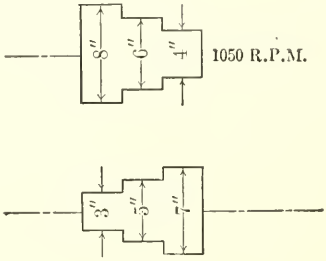


Fig. 50.

From Fig. 50 find the three speeds of the lower pulley. Ans. 600, 1260, 2800.

3. Pulleys:

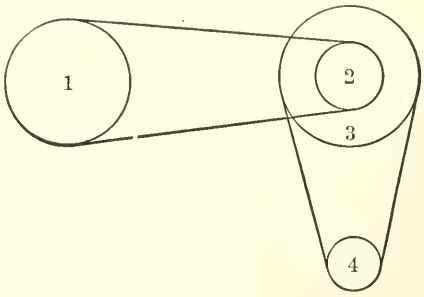


Fig. 51.

Pulley 1 is a 16-in. pulley and runs at 125 R.P.M. Pulley 2 is a 10-in. pulley.

Pulley 3 is an 18-in. pulley.
Pulley 4 is an 8-in. pulley.
Pulleys 2 and 3 are on the same shaft.
Find the R.P.M. of pulley 4. Ans. 450.

4. Gears:

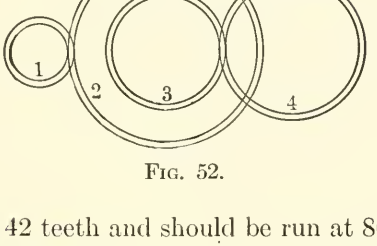


Fig. 52.

Gear 1 has 42 teeth and should be run at 800 R.P.M.
Gear 2 has 96 teeth.
Gear 3 has 48 teeth.
Gear 4 is on a shaft running at 300 R.P.M. How many teeth has gear 4? Ans. 56.

5. Dynamo under a passenger car:

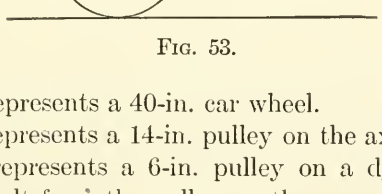


Fig. 53.

Circle 1 represents a 40-in. car wheel.
Circle 2 represents a 14-in. pulley on the axle of the car.
Circle 3 represents a 6-in. pulley on a dynamo and is driven by a belt from the pulley on the car axle.
Find the R.P.M. of the dynamo when the car is running at 30 miles per hour. Ans. 588.

CHAPTER VII

ELECTRICAL FORMULAS INVOLVING SQUARES AND SQUARE ROOTS

72. Power in a Direct Current Circuit. In a circuit carrying direct current,

$$P = EI = I^2R = \frac{E^2}{R}, \quad E = IR$$

where, P = power in watts;
E = E.M.F. in volts;
I = current in amperes;
R = resistance in ohms.

EXERCISE 1

Find the missing terms:

	P	E	I	R	Ans.
1.		110	.85		93.5, 129.41.
2.		220	.425		93.5, 517.64.
3.			1.2	220	316.8, 264.
4.		110		225	53.77, 488.
5.		220		32	1512.5, 6.875.
6.			.55	200	60.5, 110.
7.	40		.38		105.26, 276.3.
8.	75			240	134.2, 558.
9.	28.125			80	47.43, 592.
10.	100	55			1.818, 30.25.

11. Three resistances of 220, 234 and 431 ohms respectively are connected in series and a pressure of 550 volts is applied. How much power is used? Ans. 341.8 watts.

12. If the above resistances are connected in parallel, how many volts must be applied to get the same power? Ans. 175.19.

13. Which uses the more power, a lamp with a resistance of 220 ohms on a 110 volt circuit or lamp on a 110 volt circuit that uses 2.24 amperes? Ans. Second one.

14. A 110 volt arc light requires 12 amperes to operate. How many watts are used? What is the resistance? Ans. 1320 watts, 9.17 ohms.

15. An arc light requires 20 amperes to operate at 45 volts. How many horse-power does it take? (1 horse power = 746 watts.) Ans. 1.206.

16. A 12 candle power lamp on a 110 volt circuit takes 0.25 ampere. How many watts per candle power are used? Ans. 2.29.

17. A generator is producing 30,000 watts at 225 volts. What is the current flowing? Ans. 133.3.

73. Force Between Two Magnets. The force between two magnet poles is expressed by the formula:

$$F = \frac{M_1 M_2}{D^2},$$

where, F = force in dynes between two magnets;

M_1 and M_2 = pole strengths of the two magnets in unit poles;

D = distance between the two magnet poles in centimeters, (1 inch = 2.54 centimeters).

EXERCISE 2

Find the missing terms:

	F	M_1	M_2	D		
1.		2	8	4	Ans.	1.
2.	4	12.5	8			5.
3.	12		3	2		16.
4.		9	7	2		15.75.
5.	10	10		12		144.
6.	20	20	16			4.
7.		4	6	1		24.

8. Change to centimeters: 3″, 4″, 12″.
Ans. 7.62, 10.14, 30.45.

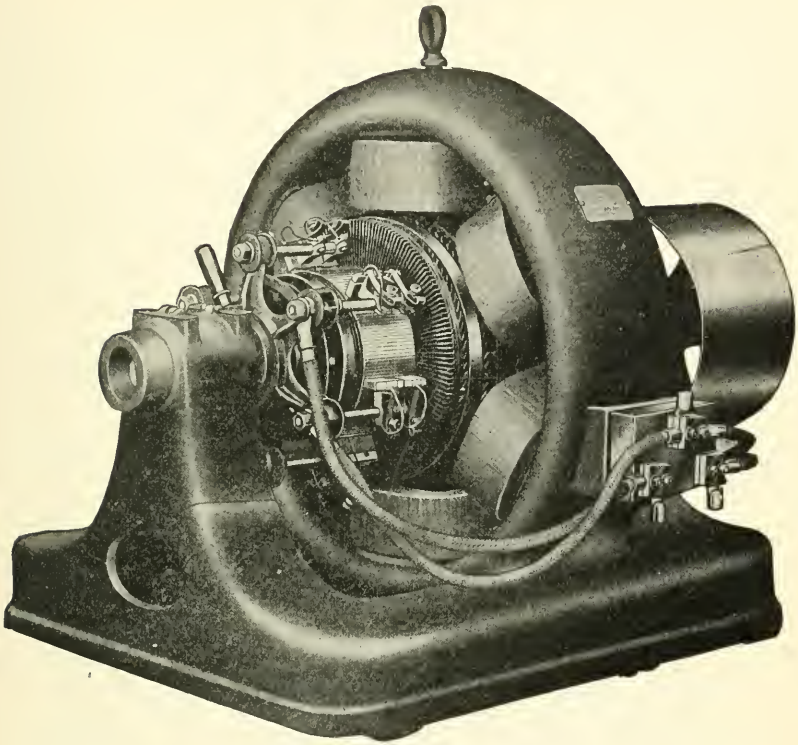
9. Change to inches: 10 cm., 50 cm., 75 cm.
Ans. 3.94″, 19.7″, 29.55″.

10. If a magnet pole of 10 unit poles is placed 4 cm. from another magnet of 24 units pole strength, what force will they exert upon each other? Ans. 15 dynes.

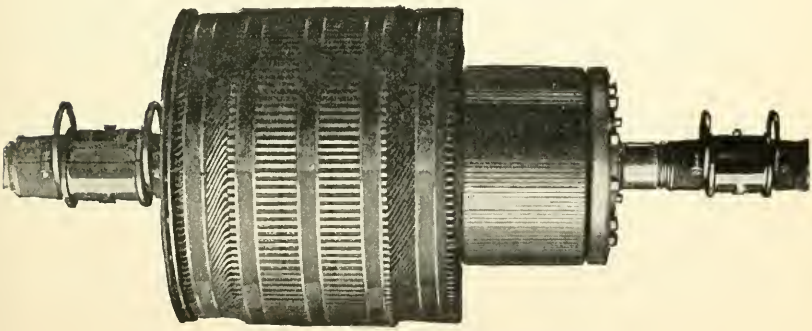
74. Horse-power of an Electric Motor. The horse-power of a motor is expressed by the formula:

$$\text{H.P.} = \frac{2\pi n T}{33000},$$

where, H.P. = horse power;
$\pi = 3.1416$;
n = R.P.M. of the motor;
T = torque in lbs. ft.



Six-Pole Direct Current Generator.



Armature of a Direct Current Generator.

EXERCISE 3

Find the missing terms:

	H.P.	n	T	Ans.
1.		1500	18.4	5.255.
2.	5	1800		14.6.
3.	1.5	1500		5.25.
4.	10		39.6	1328.
5.	8.5	1650		27.1.
6.	6.454	1200		28.25.

7. A motor running at 1040 R.P.M. develops a torque of 931 lbs. ft. Find the horse power? Ans. 185.

8. A motor is to run at 1680 R.P.M. If it has a 12″ pulley and is to furnish a pull of 200 pounds on the belt, what horse power will be required? Ans. 32.

9. A 10 H.P. motor with an 8″ pulley is to drive a 30″ pulley at 480 R.P.M. Find the torque. Ans. 29.18.

10. An 80 horse power motor developing a torque of 1231 lbs. ft. and having a 10″ pulley drives a 48″ pulley. Find the R.P.M. of the 48″ pulley. Ans. 71.

11. An electric car has a gear ratio of 14 teeth on the motor sprocket to 65 teeth on the axle sprocket. The wheels are 33″. The car has a 20 H.P. motor which gives a torque of 84.50 lbs. ft. Find the maximum speed of the car if the motor is 75 per cent efficient. Ans. 19.7.

75. Voltage of a D. C. Generator. Voltage for direct current generators is expressed by the formula:

$$E = \frac{N\phi Z p}{10^8 p' 60},$$

where, E = E.M.F. in volts;

Z = number of active conductors on the armature;

ϕ = flux in maxwells;

p = number of poles;

n = number of revolutions per minute;

p' = number of brush arms.

EXERCISE 4

Find the missing numbers:

	E	Z	ϕ	n	p	p'	Ans.
1.		200	1000000	100	2	2	70.
2.	220	226		1500	2	2	3020000.
3.	116	24		2400	8	2	4840000.
4.	120	200	1671000		2	2	2160.
5.	1040	2400	2600000	1200		2	2.
6.	112		400000	2400	4	4	700.
7.	1040	260	4000000	1500	8		2.

8. A generator has a speed of 1980 R.P.M., an E.M.F. of 50 volts, 4 poles, 200 turns, and 4 brushes. What will be the flux? Ans. 1515000.

9. How many poles will be required in a generator for a voltage of 520, 2000 surface conductors, flux of 1300000 maxwells and a speed of 1200 R.P.M. and 4 brush arms? Ans. 4.

76. Murray Loop Formula. The formula

$$\frac{R_2}{R_1} = \frac{X}{L - X},$$

is used to find the distance from a station along a telephone

line to the place where a wire is grounded; thus in Fig. 54,

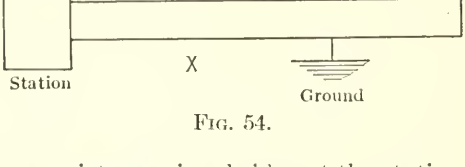


FIG. 54.

R_1 and R_2 are resistances in a bridge at the station:

L = total length of the line and return;

X = distance from the station to the place where the wire is grounded.

EXERCISE 5

Find the missing terms:

	R_1	R_2	L	X	
1.	75	50	25 miles		Ans. 10.
2.	84	20	163 miles		31.35.
3.	128	8	10 miles		.588.

4. Find the formula for X in terms of R_1, R_2, and L.

Ans. $X = \dfrac{R_2 L}{R_1 + R_2}$.

77. Designing an Inductance Coil. The formula

$$L = \frac{4\pi^2 n^2 r^2 u}{10^9 l} \quad \text{or} \quad \frac{1.26\, n^2 u A}{10^8 l},$$

is given for finding the inductance of a coil;

L = inductance in henries;

n = number of turns;

r = radius of the coil in centimeters;

u = permeability of the core $\begin{cases} = 1 \text{ for air} \\ = 1500 \text{ for iron.} \end{cases}$

A = cross section area of the core.

EXERCISE 6

1. Find the inductance of the primary coil of a transformer having 400 turns if the iron core has a cross section area of 300 sq. cm. and is 60 cm. long.

Ans. 15.1 henries.

2. Design the dimensions of an inductance coil wound on an iron core that will have an inductance of 25 henries and be of good shape.

78. Designing a Condenser. The following formula is given for the capacity of a condenser

$$C_{mf}=\frac{885\,Ka(n-1)}{10^{10}d};$$

C_{mf} = capacity of a condenser in microfarads;
K = dielectric constant;
a = area of one plate;
n = number of metal plates;
d = thickness of dielectric plate in centimeters.

EXERCISE 7

1. An aluminum and air condenser has 25 aluminum plates of diameter 12 cm., separated by 2 mm. of air. Find the capacity if $K=1$ for air. Ans. .0012.

2. A condenser is to be made of mica and tinfoil to have a capacity of .32 microfarad. The sheets are 20 cm. square and the mica is .05 cm. thick, K for mica $=6$. How many sheets are needed? Ans. 76.

3. Design the dimensions of a condenser of tinfoil and glass that will have a capacity of 0.002 microfarad. Assume that the glass available has a thickness of 0.25 cm.

$K=6$ for glass.

EXERCISE 8

General Formulas. Evaluate the following formulas:

1. $x=\dfrac{-b-\sqrt{b^2+4ac}}{2a}$, when $a=1,\ b=4,\ c=5$.

Ans. 1.

2. $x=\dfrac{-b-\sqrt{b^2+4ac}}{2a}$, when $a=1,\ b=4,\ c=5$.

Ans. -5.

3. $x=\dfrac{-b+\sqrt{b^2+4ac}}{2a}$, when $a=3,\ b=-14,\ c=-6$.

Ans. 4.1893 and 0.4774.

4. $b=\sqrt{a^2+c^2-2a'c}$, when $a=7,\ c=5,\ a'=3$.

Ans. 6.633.

5. $A=\sqrt{s(s-a)\,(s-b)\,(s-c)}$, when $a=16,\ b=12,$ $c=20$ and $s=\frac{1}{2}(a+b+c)$. Ans. 96.

6. $A=\sqrt{s(s-a)\,(s-b)\,(s-c)}$, when $a=22,\ b=30,$ $c=26$ and $s=\frac{1}{2}(a+b+c)$. Ans. 278.5.

CHAPTER VIII

QUADRATIC EQUATIONS

79. Definitions. Some formulas and problems require the use of an unknown letter raised to the second power, giving an equation containing x^2 or y^2, etc. An equation that contains an unknown in the second power as the highest power of the unknown is a *quadratic equation.*

If a quadratic equation contains only the second power, of the unknown, it can be put in the form $x^2 = 25$. (**Axiom:** *If two expressions are equal their square roots are equal.*)

Therefore, since

$$x^2 = 25$$
$$x = \pm 5 \ (\pm \text{ is read } + \text{ or } -)$$

Observe that:

1. *The sign* $\pm$ *means that both* $+5$ *and* -5 *check the equation.*

2. *The square root of a number may be positive or negative because* $(+5)^2 = 25$ *and* $(-5)^2 = 25$.

3. *A quadratic equation has two answers.*

EXERCISE 1

Solve the following equations:

1.	$x^2 = 121$.	Ans. $x = \pm 11$.
2.	$x^2 = 50$.	$x = \pm 7.071$.
3.	$R^2 = \frac{3}{4}$.	$R = \pm .866$.
4.	$d^2 = \frac{2}{3}\frac{5}{6}$.	$d = \pm \frac{5}{6}$.

5. $y^2=\frac{7}{8}$. Ans. $y=\pm 935$.
6. $y^2-12=37$. $y=\pm\ 7$.
7. $5x^2=180$. $x=\pm\ 6$.
8. $3x^2+13=160$. $x=\pm\ 7$.

80. Quadratic Equations with Both First and Second Powers. A quadratic equation which contains the first and second powers of the unknown cannot be solved by the above method. For example, in $x^2+6x=40$, the square root of x^2+6x cannot be found. The form of the equation can be changed so that the square roots of both members can be found.

EXERCISE 2

Expand the following:

1. $(x+1)^2$. Ans. x^2+2x+1.
2. $(x+2)^2$.
3. $(x-1)^2$.
4. $(x-4)^2$.
5. $(x+5)^2$.
6. $(x+\frac{1}{3})^2$.
7. $(x+\frac{3}{4})^2$.
8. $(x-\frac{1}{2})^2$.

Observe that:

1. *The answers to all the problems in Exercise 2 contain three terms. An x^2 term, an x term, and a numerical term.*

2. *The x^2 term and the numerical term are perfect squares and the x term is twice the product of the two terms of the problem. (For example, in $(x-4)^2=x^2-8x+16$, x^2 and 16 are the squares of x and 4, while $-8x$ is $2x(-4)$.)*

81. Trinomial Squares. An expression having two terms which are perfect squares and the other term twice the product of the square roots of these two terms is a *trinomial square.* The square root of a trinomial square can be found. The process is the reverse of the method used in Exercise 2.

Rule. *To find the square root of a trinomial square take the square root of the two perfect square terms and connect them by the sign of the other term.*

Example 1. $\sqrt{x^2+6x+9}=\pm(x+3)$.

Example 2. $\sqrt{x^2-10x+25}=\pm(x-5)$.

EXERCISE 3

Find the square root of the following; give positive results only:

1. x^2+2x+1. Ans. $x+1$.
2. x^2-4x+4. $x-2$.
3. x^2-6x+9.
4. $x^2+14x+49$.
5. $y^2-16y+64$.
6. $x^2+20x+100$.
7. $R^2-R+\frac{1}{4}$.
8. $a^2-\frac{4}{3}a+\frac{4}{9}$.

82. To Complete a Trinomial Square. A quadratic equation of the form $x^2+6x=40$ can be changed to an equation having a trinomial square in the first member by adding 9 to both members, thus:

$$x^2+6x \quad =40$$

$$x^2+6x+9=49$$

The number 9 which must be added to x^2+6x to make it a trinomial square is the square of $\frac{1}{2}$ of 6 or 9. (This follows from Exercises 2 and 3.)

Rule. *To change an expression of the form x^2+ax into a trinomial square add to it the square of $\frac{1}{2}$ the coefficient of x.*

Example 1. Find the number which will make x^2+10x a trinomial square.

Solution. The coefficient of x is 10, $\frac{1}{2}$ of $10=5$, $5^2=25$. Therefore 25 added to x^2+10x will make it a trinomial square.

Example 2. Find the number which will make x^2-7x a trinomial square.

Solution The coefficient of x is 7, $\frac{1}{2}$ of $7=\frac{7}{2}$, $(\frac{7}{2})^2=\frac{49}{4}$. Therefore $\frac{49}{4}$ added to x^2-7x will make it a trinomial square.

EXERCISE 4

Find the number which will add to the following to make trinomial squares:

	Ans.
1. x^2+2x.	1.
2. x^2-6x.	9.
3. x^2+10x.	25.
4. x^2+5x.	$\frac{25}{4}$.
5. x^2-9x.	$\frac{81}{4}$.
6. x^2-12x.	36.

83. Solution of Quadratic Equations by Completing the Square.

Example 1. $x^2+6x=40$

$x^2+6x+9=49$ (Adding 9 to both members to make the first member a trinomial square)

$x+3=\pm 7$ (Extracting the square root of both members)

Then $x+3=+7$ and $x+3=-7$

$x=4$ $x=-10$

Check. $4^2+6.4=40$ $(-10)^2+6(-10)=40$

$16+24=40$ $100-60=40$

$40=40$ $40=40$

Example 2. $2x^2-6x=12+4x$

$2x^2-10x=12$

$x^2-5x=6$ (The coefficient of x^2 must be made equal to 1)

$x^2-5x+\frac{25}{4}=\frac{49}{4}$ (Adding $\frac{25}{4}$ to both members to make the first member a trinomial square)

$x-\frac{5}{2}=\pm\frac{7}{2}$

Then $x-\frac{5}{2}=+\frac{7}{2}$ and $x-\frac{5}{2}=-\frac{7}{2}$

$x=\frac{12}{2}$ $x=-\frac{2}{2}$

$x=6$ $x=-1$

Observe that:

1. *A quadratic equation has two answers.*
2. *Both answers must check.*
3. *Before completing the square the first term must have the coefficient* 1.

EXERCISE 5

Solve the following equations:

	Ans.
1. $x^2+4x=45$.	$x=5$ or -9.
2. $x^2+6x=27$.	$x=3$ or -9.
3. $x^2-5x=24$.	$x=8$ or -3.
4. $2x^2-7x=30$.	$x=6$ or $-2\frac{1}{2}$.
5. $2x^2-7x=34$.	$x=6.229$ or 2.729.
6. $3x^2+7x=8$.	$x=1$ or $-2\frac{2}{3}$.

7. $x^2+x^2+10x=22+3x$. Ans. $x=2$ or $-5\frac{1}{2}$.

8. $\frac{x^2}{2}-\frac{7x}{8}=\frac{51}{4}$. $x=6$ or $-4\frac{1}{4}$.

84. Solution of Quadratic Equations by Formula. Any quadratic equation can be expressed in the form $ax^2+bx=c$, where a, b and c are general numbers. If the equation $ax^2+bx=c$ is solved for x by completing the square, it will be found that

$$x=\frac{-b\pm\sqrt{b^2+4ac}}{2a}$$

The above result should be memorized and used as a formula for solving other quadratic equations of the form $ax^2+bx=c$.

Observe that *a is the coefficient of x^2, b is the coefficient of x and c is the term that does not contain x.*

Problems.

1. $ax^2+bx=c$.

Solutions.

$$x=\frac{-b\pm\sqrt{b^2+4ac}}{2a}$$

2. $2x^2+5x=18$ $\left|\begin{array}{l} a=2 \\ b=5 \\ c=18 \end{array}\right|$

$$x=\frac{-5\pm\sqrt{5^2+4\cdot 2\cdot 18}}{2\cdot 2}$$

Simplifying:

$$x=\frac{-5\pm\sqrt{25+144}}{4}$$

$$x=\frac{-5\pm\sqrt{169}}{4}$$

$$x=\frac{-5\pm 13}{4}$$

$$x=\frac{8}{4} \quad \text{or} \quad \frac{-18}{4}$$

$$x=2 \quad \text{or} \quad -4\tfrac{1}{2}$$

3. $3x^2-4x=15$ $\left|\begin{array}{l} a= \quad 3 \\ b=-\ 4 \\ c= \quad 15 \end{array}\right|$ Substitute in the formula.

4. $x^2-2x-5=45-3x.$

$x^2-5x=50.$ (Arranging in the standard form.)

$\left|\begin{array}{l} a= \quad 1 \\ b=-\ 5 \\ c= \quad 50 \end{array}\right|$ Substitute in the formula.

EXERCISE 6

Solve by the formula method:

1.	$2x^2+5x=3.$	Ans. $-3,\ +.5.$
2.	$5x^2+3x=2.$	$\frac{2}{5},\ -1.$
3.	$x^2+4x=5.$	$1,\ -5.$
4.	$x^2-2x=15.$	$5,\ -3.$
5.	$x^2=4x+12.$	$6,\ -2.$
6.	$2x^2-3x=18.$	$3.84,\ -2.34.$
7.	$7x^2-4x-8=10.$	$1.889,\ -1.318.$
8.	$5x^2-6x=41.$	$3.526,\ -2.326.$
9.	$\frac{5}{3x-7}=\frac{x}{x+84}.$	$14,\ -10.$
10.	$x^2+3x=9.$	$-4.854,\ 1.854.$
11.	$3x^2+5x=2.$	$-2,\ \frac{1}{3}.$

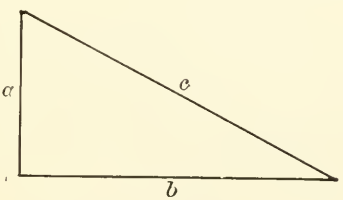


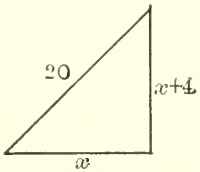
Fig. 55.

85. Application of the Quadratic Equation to the Right Triangle. If the hypotenuse of a right triangle (Fig. 55)

is c and the sides are a and b the relation between the sides, and the hypotenuse is expressed by the formula,

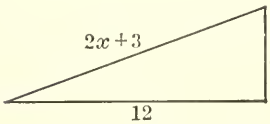
$$c^2 = a^2 + b^2.$$

EXERCISE 7

1. 

Find the sides. Ans. 12, 16.

Fig. 56.

2. 

Find the sides. Ans. 5, 13.

Fig. 57.

3. One side of a right triangle is 2 more than the other side and the hypotenuse is 10. Find the sides. Ans. 6, 8.

Hint. Let x = one side
then $x+2$ = the other side

4. One side of a rectangular lot is 31 rods more than the other side and the area is 360 square rods. Find the dimensions. Ans. 9, 40.

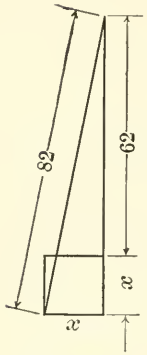


Fig. 58.

5. Find the side of the square in Fig 58.

Ans. 18.

6. Find the inside and outside radius of a ring that has an area of 34.5576 square inches and a thickness of 1 inch.

Ans. 5, 6.

7.

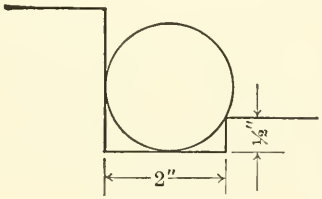


Fig. 59.

Find the size plug that will fit as shown in Fig. 59.

Ans. Radius = 1.0858.

CHAPTER IX

SIMULTANEOUS EQUATIONS

86. Definition. In some problems which involve two or more unknown quantities it is often difficult to express both unknowns in terms of one letter. Such problems can be solved conveniently by using two or more unknown letters.

For example, The sum of two numbers is 29 and the difference is 11. Find the numbers.

Let x = one of the numbers
y = the other number

Then $x+y=29$
$x-y=11$

Neither one of these two equations can be solved independently; they must be solved together. Two or more equations involving two or more unknowns which must be solved together are called **simultaneous equations.**

87. Solution of Simultaneous Equations. Two simultaneous equations can be solved by eliminating one of the unknowns.

(*a*) *Elimination by Addition or Subtraction:*

Example 1. The two simultaneous equations above can be solved by adding the first member of the one to the first member of the other, and the second member to second

member. The sums will be equal. (**Axiom:** If equals are added to equals the sums are equal.)

$$x+y=29 \quad \ldots\ldots\ldots \quad (1)$$
$$x-y=11 \quad \ldots\ldots\ldots \quad (2)$$
$$2x=40 \quad \text{Adding (1) and (2)} \quad (3)$$
$$x=20$$
$$20+y=29 \quad \text{Substituting value of } x$$
$$y=9 \quad \text{in (1),}$$

Check.
$$20+9=29$$
$$29=29$$
$$20-9=11$$
$$11=11$$

NOTE. *When one unknown is found the other may be found by substituting the value found in either of the original equations. To check simultaneous equations the values found must check both equations.*

Example 2.
$$x+3y=8 \quad \ldots\ldots\ldots \quad (1)$$
$$2x+y=11 \quad \ldots\ldots\ldots \quad (2)$$
$$2x+6y=16 \quad \text{Multiplying both members of (1) by 2.} \quad (3)$$

$$-5y=-5. \quad \text{Subtracting (3) from (2).}$$
$$y=1.$$
$$x+3=8. \quad \text{Substituting in (1).}$$
$$x=5.$$

Check.
$$5+3=8.$$
$$8=8.$$
$$10+1=11.$$
$$11=11.$$

Example 3.

$$7x+3y=16. \quad \ldots\ldots\ldots \quad (1)$$
$$3x+2y=\ 9. \quad \ldots\ldots\ldots \quad (2)$$
$$14x+6y=32. \quad (1)\times 2. \quad \ldots\ldots \quad (3)$$
$$9x+6y=27. \quad (2)\times 3. \quad \ldots\ldots \quad (4)$$
$$5x=\ 5. \quad (3)-(4).$$
$$x=\ 1.$$
$$7+3y=16. \quad \text{Substituting in (1),}$$
$$3y=\ 9.$$
$$y=3.$$

Check.

$$7+9=16.$$
$$16=16.$$
$$3+6=\ 9.$$
$$9=\ 9.$$

Observe that:

1. *Either unknown may be eliminated.*

2. *To eliminate an unknown it may be necessary to multiply both members of one equation by some number, and in some problems it may be necessary to multiply both members of both equations by some number. Both equations do not have to be multiplied by the same number.*

3. *In checking simultaneous equations the values found must check both equations.*

EXERCISE 1

Solve the following simultaneous equations:

1. $2x+3y=16.$
   $2x-\ y=\ 8.$ Ans. $x=5,\ y=2.$
2. $2x+\ y=\ 4.$
   $3x-\ y=21.$ $x=5,\ y=-6.$
3. $8x+\ y=\ 7.$
   $11x+2y=28.$ $x=-2\frac{4}{5},\ y=29\frac{2}{5}.$

4. $x-2y=-12.$
$4x-\ y=\ 1.$ Ans. $x=2,\ y=7.$

5. $3x+5y=33.$
$4x-2y=18.$ $x=6,\ y=3.$

(*b*) *Elimination by Substitution.* Some simultaneous equations can be solved more efficiently by another method called *substitution.* In this method, solve either equation for either unknown in terms of the other unknown (choose the one giving the simplest solution), and substitute the value found in the other equation.

Example 1.
$$x+y=5 \quad \ldots\ldots\ldots\ldots \quad (1)$$
$$2x+3y=13 \quad \ldots\ldots\ldots\ldots \quad (2)$$
$$x=5-y \quad \text{Solving (1) for } x. \quad \ldots \quad (3)$$
$$2(5-y)+3y=13 \quad \text{Substituting in (2),}$$
$$10-2y+3y=13$$
$$y=3$$
$$x+3=5$$
$$x=2$$

Check.
$$2+3=5$$
$$5=5$$
$$2\cdot 2+3\cdot 3=13$$
$$4+9=13$$
$$13=13$$

Example 2.
$$3x+2y=25 \quad \ldots\ldots\ldots\ldots \quad (1)$$
$$2x-5y=4 \quad \ldots\ldots\ldots\ldots \quad (2)$$
$$2x=4+5y \quad \text{From (2),}$$
$$x=\frac{4+5y}{2}$$
$$\frac{3(4+5y)}{2}+2y=25 \quad \text{Substituting in (1),}$$
$$3(4+5y)+4y=50$$

$$12+15y+4y=50$$
$$19y=38$$
$$y=2$$
$$2x-10=4$$
$$2x=14$$
$$x=7$$

EXERCISE 2

Solve by the substitution method:

1. $2x+3y=26.$
   $x-5y=0.$ Ans. $x=10,\ y=2.$
2. $3y=5x-18.$
   $x=\frac{3y}{2}.$ $x=6,\ y=4.$
3. $x=2y+3.$
   $x=7y-12.$ $x=9,\ y=3.$
4. $y-3x=2.$
   $x^2+y^2=4.$ $x=0,\ y=2$ or $x=-1\frac{1}{5},\ y=-1\frac{3}{5}.$
5. $4x^2+y^2=5.$
   $y^2=8x.$ $x=\frac{1}{2},\ y=2$ or $x=\frac{1}{2},\ y=-2.$

CHAPTER X

THE GRAPH

88. Plotting a Graph. The number of revolutions per minute, that will give a cutting speed of 45 ft. per minute to the following size stock, may be found and the results expressed in tabular form thus:

	Diameter of Stock.	R.P.M.
1.	$\frac{1}{2}''$	343
2.	$\frac{3}{4}''$	229
3.	$1''$	172
4.	$1\frac{1}{4}''$	137
5.	$1\frac{1}{2}''$	114
6.	$2''$	86
7.	$2\frac{1}{2}''$	69
8.	$3''$	57

These results may also be represented graphically by plotting the diameters along a horizontal line (called an axis) and the R.P.M. along a line perpendicular to this line (also an axis), as in Fig. 64.

In Fig. 64 point 1 corresponds to a diameter of $\frac{1}{2}$ in. and 343 R.P.M. Similarly point 2 may be located, etc.

Observe that:

1. *All of the points are connected by a smooth regular curve.*

2. *All points on the curve represent some corresponding diameter and R.P.M. that will give a cutting speed of 45 ft. per minute.*

3. *When either diameter of stock or R.P.M. is given, the other can be found from the figure.*

The horizontal axis is called the **X-axis** and the vertical

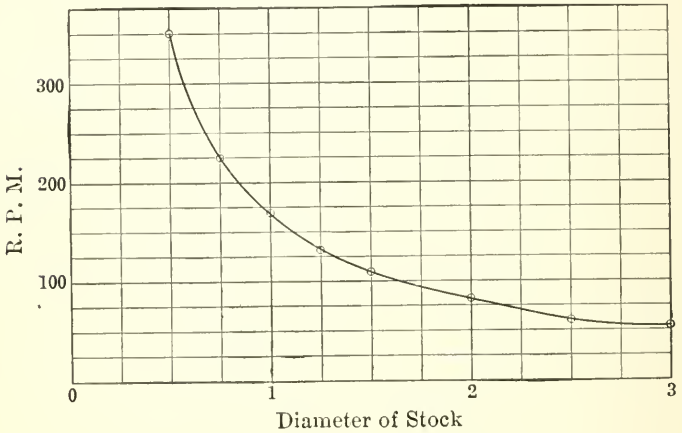


Fig. 60.

axis is called the **Y-axis.** The curve connecting the points located is a **graph.**

Example. From Fig. 60, find the R.P.M. necessary to give a $2\frac{1}{4}$-in. stock a cutting speed of 45 ft. per minute.

Solution. Find $2\frac{1}{4}$ in. on the X-axis. Find the point on the graph directly above this point and read the R.P.M. on the Y-axis (75) corresponding to this point.

EXERCISE 1

From Fig. 60 find the R.P.M. necessary to give a cutting speed of 45 ft. per minute to the following stock:

	Diameter of Stock.	R.P.M.
1.	$2\frac{3}{4}''$	...
2.	$2\frac{1}{8}''$	...
3.	$1\frac{3}{8}''$	...
4.	$\frac{7}{8}''$	...

EXERCISE 2

1. Make a table of diameters and R.P.M. as on page 109, for a cutting speed of 85 ft. per minute. (The slide rule can be used to advantage here.)

2. Plot on squared paper a graph of the data computed in Problem 1.

NOTE. The value of the units on the squared paper must be so chosen that the curve will all lie on the sheet of paper, and the units must be sufficiently large to permit reading nearly accurate values from the curve. (The larger the curve the more accurate the results.)

3. From the curve of Problem 2, find the R.P.M. necessary to give the following stock a cutting speed of 85 ft. per minute:

	Diam.	R.P.M.
(*a*)	$\frac{7}{8}''$	...
(*b*)	$1\frac{3}{8}''$	...
(*c*)	$2\frac{1}{8}''$	...
(*d*)	$2\frac{3}{4}''$	...

4. Students having use in the shop for a set of these curves should plot a curve for several other cutting speeds in common use. These curves can all be plotted on the same scale and axes.

EXERCISE 3

1. A storage battery is discharged in 100 minutes and the voltage furnished by the battery measured at different times during the discharge. Following is a table of the data read:

Time (in minutes).	Voltage.
0	6
2	5.9
4	5.8
10	5.7
12	5.7
17	5.6
22	5.5
27	5.5
32	5.5
37	5.4
42	5.3
52	5
62	4.7
72	4.3
82	3.7
92	2.7
94	2.5
97	1.9
100	1.3

(*a*) Plot a graph of the discharge, laying off the time on the X-axis and the voltage on the Y-axis. Connect the points plotted by a smooth curve.

(*b*) Read from the curve the voltage at the end of 30 minutes.

(*c*) What does the shape of the curve indicate?

EXERCISE 4

1. Compute the weight per foot of round steel, and fill in the following table (1 cu. in. of steel weighs .28 lb.):

	Diam. (Inches).	Weight.
1.	$\frac{1}{8}$	....
2.	$\frac{1}{4}$	....
3.	$\frac{3}{8}$	....
4.	$\frac{1}{2}$	....
5.	$\frac{3}{4}$	....
6.	1	....
7.	$1\frac{1}{4}$	....
8.	$1\frac{1}{2}$	....
9.	2	....
10.	$2\frac{1}{2}$	....
11.	3	....

2. Plot a curve from the above data. Read from the curve the missing values in the following table:

	Diameter.	Weight.
(*a*)	$\frac{7}{8}$	....
(*b*)	...	13.4
(*c*)	$2\frac{3}{4}$	....

89. Coordinates. Many problems arise in which it is necessary to plot or locate points with reference to one another. A point is generally located, as in the previous exercises, by giving its distance from each of two perpendicular lines. Such a system is a **rectangular coordinate system.** The two distances are the **coordinates** of the point. The two perpendicular lines are **axes:** the horizontal line is the X-axis, and the vertical line is the Y-axis. The

point where the axes meet is the **origin**. The axes are drawn and lettered as in Fig. 61.

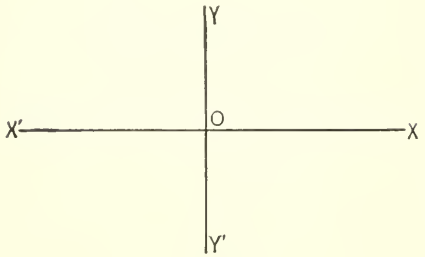


Fig. 61.

The horizontal distance of a point from the origin is the **abscissa** of the point. The vertical distance of a point from the origin is the **ordinate** of the point. The abscissa is usually represented by x and the ordinate by y.

90. Location of Points. Any point is represented by the symbol (x, y) and a particular point is represented by the symbol $(2, 7)$, $(-3, 5)$, etc., the abscissa always being given first. Negative abscissas are plotted to the left of the origin and negative ordinates below the origin.

To locate a point, lay off the axes (squared paper is the most convenient to use); then lay off the abscissa along the X-axis, and lay off the ordinate perpendicular to the X-axis at this point.

Example 1. Locate the point $(3, 5)$.

Solution. Measure 3 from the origin along the X-axis to the right, then 5 above this point. This is the point $(3, 5)$, as shown in Fig. 62.

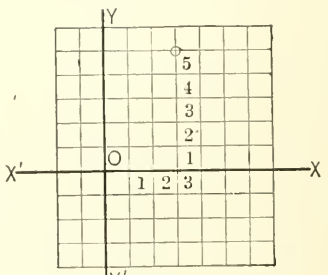


Fig. 62.

Example 2. Locate the point $(4, -5)$.

Solution. Measure 4 from the origin along the X-axis to the right then 5 below this point, as in Fig. 63.

EXERCISE 5

Locate the following points:

1. $(5, 3)$.
2. $(-4, 2)$.
3. $(2, 7)$.
4. $(-5, -2)$.
5. $(4, -3)$.
6. $(-4, 0)$.
7. $(0, 4)$.
8. $(3\frac{1}{2}, 4\frac{3}{4})$.

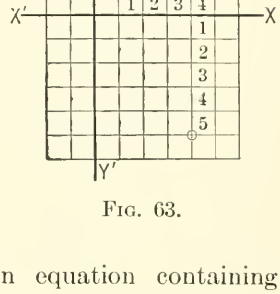


Fig. 63.

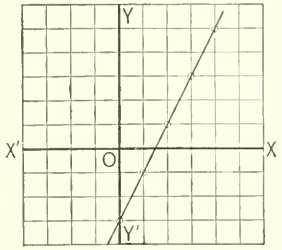
91. Graph of an Equation. An equation containing two unknowns, as $x+y=7$, cannot be solved for a value of x and y, since many different pairs of values would check, as $x=3$, $y=4$, or $x=2$, $y=5$, etc. By plotting several points representing corresponding values of x and y and connecting these points, a line is obtained which expresses the relation between x and y in the equation. The line is the **graph of the equation.**

Example 1. Construct the graph of the equation

$$y=2x-3.$$

Solution. Find several pairs of corresponding values of x and y. This can be done most efficiently by assuming values of x and computing by evaluation the corresponding

values of y. For example, when $x=2$, then $y=2\cdot 2-3=1$. Record the values found in a table thus:



x	y
0	-3
1	-1
2	1
3	3
4	5

Fig. 64.

Plot the points on squared paper, as in Fig. 64.

The line connecting the points is a graph of the equation $y=2x-3$, and represents the relation between x and y. The coordinates of every point on the line satisfy the equation, and every point whose coordinates satisfy the equation lies on the line. Any number of corresponding values of x and y can be read from the figure.

EXERCISE 6

Draw the graph of the following equations:

1. $y=3x$.
2. $y=2x-1$.
3. $x+y=5$. (Solve for y first.)
4. $2x+3y=13$.

92. Graphs which Are Not Straight Lines. All the above graphs are straight lines, and when a few points are located the whole graph can be drawn. If the graph of an equation is not a straight line, more points will have to be plotted to determine the shape of the graph. Connect the points by a smooth curve. (A French curve will assist in this.)

EXERCISE 7

Draw the graphs of the following curves:

1. $y=2x^2+1$.

NOTE. $x=+3$ or -3 will give the same value of y, therefore x has two values giving two points one above the X-axis and the other below the X-axis.

2. $x^2+y^2=25$.
3. $x^2-y^2=16$.
4. $4x^2+9y^2=36$.

93. Graphs of Simultaneous Equations. The graph of the equation $2x=3y-5$ is the line AB, Fig. (65). The graph of the equation $3x+2y=12$ is the line CD, Fig. (65).

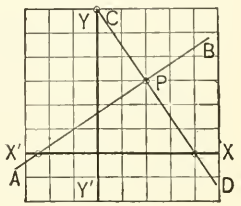


FIG. 65.

Observe that:

1. *The coordinates of all points on the line AB check the equation*

$$2x=3y-5.$$

2. *The coordinates of all points on the line CD check the equation*

$$3x+2y=12.$$

3. *The coordinates of the point P (2, 3) check both equations.*

4. *The coordinates of the point P are the same as the values of x and y when the two equations are solved simultaneously.*

94. Solution of Simultaneous Equations by Graphs. If the two equations are graphed, using the same axes, the coordinates of the points of intersection of the two graphs check both equations and are the values of the unknowns found when the equations are solved simultaneously.

Example 1. Solve the equations $x^2+y^2=25$ and $4x=3y$ by plotting the graphs and reading the coordinates of the points of intersection of the graphs, and check by solving as simultaneous equations.

From Fig. 66 it will be seen that there are two points of intersection P (3, 4) and P' $(-3, -4)$.

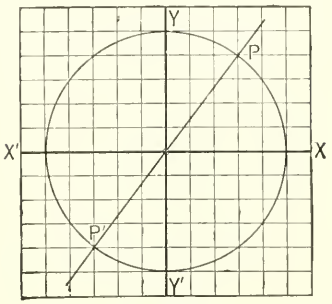


FIG. 66.

Check.

$$x^2+y^2=25$$
$$4x=3y$$
$$x=\tfrac{3}{4}y$$
$$(\tfrac{3}{4}y)^2+y^2=25$$
$$\frac{9y^2}{16}+y^2=25$$
$$9y^2+16y^2=400$$
$$25y^2=400$$
$$y^2=16$$
$$y=\pm 4$$
$$x=\frac{3y}{4}=\frac{3(\pm 4)}{4}=\pm 3$$

EXERCISE 8

Solve the following equations graphically and check by solving simultaneously:

1. $2x+y=6.$
   $x+3y=13.$
   Ans. $x=1,\ y=4.$
2. $3x+2y=12.$
   $y=4x-5.$
   $x=2,\ y=3.$
3. $x^2+y^2=36.$
   $x+y=4.$
   $x=5.742,\ y=-1.742$
   or $-x=1.742,\ y=5.742.$
4. $4x^2+9y^2=72.$
   $3y=2x.$
   $x=3,\ y=2,$
   or $x=-3,\ y=-2.$

CHAPTER XI

GEOMETRY

95. Angles. If the line OA (Fig. 67) revolves about O as center, to the position OB, the two lines form an **angle.** The point O is the **vertex** of the angle. The lines OA and OB are the **sides** of the angle. An angle is read by reading the letter at the vertex between the letters at the ends of the sides. Thus, the angle in Fig. 67 is read angle AOB. The size of the angle is independent of the length of the sides, and is measured by the fractional part of a revolution made in turning from OA to OB.

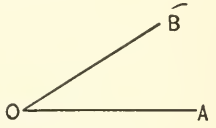


FIG. 67.

96. Right Angle. If the line OB makes *one-fourth* of a complete revolution, as Fig. 68, the angle AOB is a **right angle.**

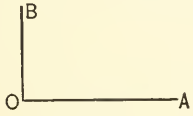


FIG. 68.

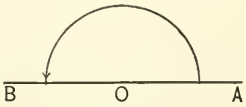


FIG. 69.

97. Straight Angle. If the line OB makes *one-half* of a complete revolution, as in Fig. 69, the angle AOB is a **straight angle.**

98. Perigon. If the line OB makes one complete revolution, the angle AOB is a **perigon.**

99. Degree. A right angle is divided into 90 equal parts called **degrees.** A degree is divided into 60 equal parts called **minutes.** A minute is divided into 60 equal parts called **seconds.** Degrees, minutes, and seconds are indicated thus: 50 degrees, 32 minutes, 24 seconds, or $50° 32' 24''$.

100. Protractor. A protractor (Fig. 70) is an instrument for measuring and constructing angles.

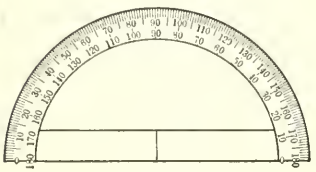


Fig. 70.

101. Supplementary Angles. If the sum of two angles is a straight angle or 180°, the angles are **supplementary angles.** Each is the supplement of the other, *i.e.*, represents what must be added to make 180°.

102. Complementary Angles. If the sum of two angles is 90°, the angles are **complementary angles.** Each angle is the complement of the other, *i.e.*, represents what must be added to make 90°.

103. Angle Theorems.

1. All right angles are equal.
2. All straight angles are equal.
3. Complements of equal angles are equal.
4. Supplements of equal angles are equal.
5. The sum of the angles about a point on one side of a straight line is equal to 180°.

6. The sum of the angles of a triangle is equal to 180°.

7. If two straight lines intersect, as in Fig. 71, the opposite or vertical angles are equal. That is,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

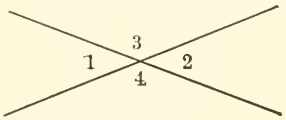


Fig. 71.

8. If two angles have their sides parallel right side to right side and left side to left side, as in Fig. 72, they are equal.

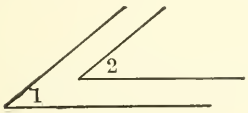


Fig. 72.

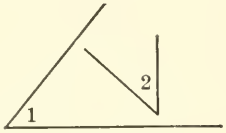


Fig. 73.

9. If two angles have their sides perpendicular, right side to right side and left side to left side, as in Fig. 73, they are equal.

104. Congruent Triangles. Triangles equal in all respects are **congruent.**

1. If two triangles have two sides and the included angle of the one equal to two sides and the included angle

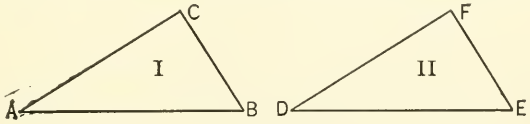


Fig. 74.

of the other, the triangles are congruent. That is, if $AC = DF$, $AB = DE$ and $\angle A = \angle D$, Fig. 74, triangles I and II are congruent.

2. If two triangles have two angles and the included side of the one equal to two angles and the included side of the other, the triangles are congruent. That is, in Fig. 75, if

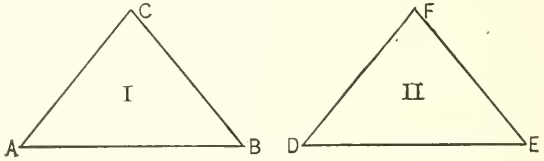


Fig. 75.

$\angle A = \angle D$, $\angle B = \angle E$ and $AB = DE$, triangles I and II are congruent.

3. If two triangles have three sides of the one equal to three sides of the other, the triangles are congruent. That is, in Fig. 76, if $AB = DE$, $AC = DF$, $BC = EF$, triangles I and II are congruent.

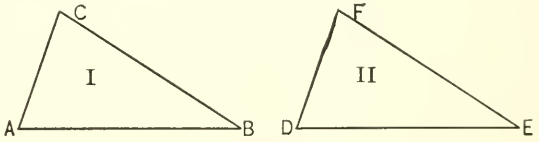


Fig. 76.

4. Corresponding parts of congruent triangles are equal.

105. Isosceles Triangle Theorems.

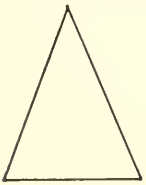


Fig. 77.

1. In an isosceles triangle (Fig. 77) the angles opposite the equal sides are equal.

2. In an isosceles triangle the bisector of the vertex angle is the perpendicular bisector of the base.

3. If two angles of a triangle are equal

the sides opposite the equal angles are equal and the triangle is isosceles.

106. Parallel Lines. Straight lines in the same plane that will not meet, however far they are extended, are parallel lines.

1. A perpendicular to one of two parallel lines (Fig. 78) is perpendicular to the other also.

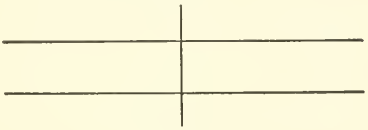


Fig. 78.

2. If two parallel lines are cut by a third straight line, (Fig. 79):

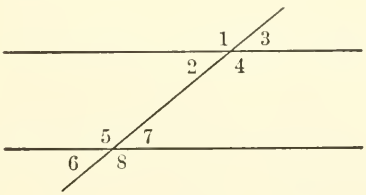


Fig. 79.

(*a*) The alternate interior angles are equal

(*i.e.*, $\angle 2 = \angle 7$ and $\angle 4 = \angle 5$).

(*b*) The alternate exterior angles are equal

(*i.e.*, $\angle 1 = \angle 8$ and $\angle 3 = \angle 6$).

(*c*) The corresponding angles are equal

(*i.e.*, $\angle 1 = \angle 5$, etc.).

(*d*) The interior angles on the same side of the third line are supplementary

(*i.e.*, $\angle 2 + \angle 5 = 180°$, etc.).

3. If two lines are cut by a third line making

(*a*) The alternate interior angles equal,

(*b*) The alternate exterior angles equal,

(*c*) The corresponding angles equal, or

(*d*) The interior angles on the same side of the third line supplementary,

the lines are parallel.

NOTE. No. 3 is the converse of No. 2.

4. Two lines perpendicular to the same straight line (Fig. 80) are parallel.

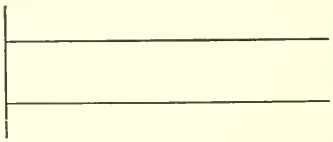


FIG. 80.

107. Quadrilaterals. A plane figure having four straight sides is a **quadrilateral.**

A **trapezium** is a quadrilateral having no two sides parallel.

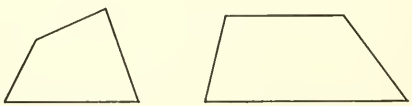


FIG. 81.

A **trapezoid** s a quadrilateral having one pair of parallel sides.

A quadrilateral having its opposite sides parallel is a **parallelogram.**

A **rectangle** is a parallelogram having four right angles.

A **square** is a rectangle having four equal sides.

A **rhombus** is a parallelogram having four equal sides and four oblique angles.

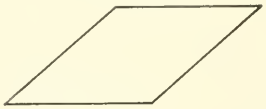


FIG. 82.

108. Parallelogram Theorems.

1. The opposite sides of a parallelogram are equal.

2. The opposite angles of a parallelogram are equal.

3. Two adjacent angles of a parallelogram are supplementary.

4. If a pair of opposite sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

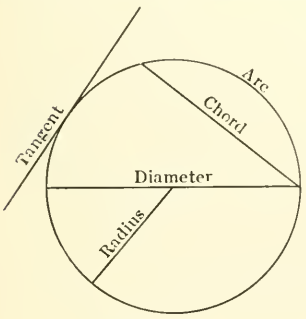


FIG. 83.

109. Circles.

1. The diameter of a circle bisects the circle.

2. In the same circle or in equal circles, radii that form equal angles at the center intercept equal arcs on the circumference, and conversely.

That is, if circle I is equal to circle II (Fig. 84), and if

$$\angle 1 = \angle 2,$$

then arc 1 = arc 2

or if arc 1 = arc 2,

then $\angle 1 = \angle 2$.

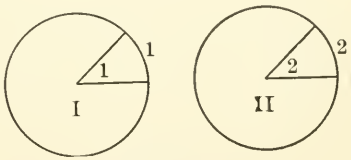


FIG. 84.

3. In the same circle or in equal circles equal chords subtend equal arcs, and conversely.

If chord AB = chord CD (Fig. 85), then arc AB = arc CD, and conversely.

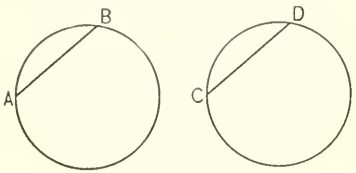


Fig. 85.

4. In the same circle or in equal circles equal chords are equal distances from the center (Fig. 86), and conversely, if $AB = CD$, then $OX = MN$, and conversely.

5. A line fulfilling any two of the following conditions fulfills the other two (Fig. 87):

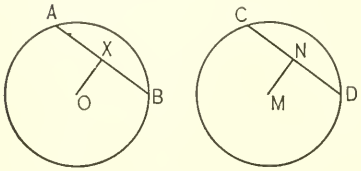


Fig. 86.

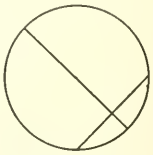


Fig. 87.

(*a*) Passes through the center of the circle.
(*b*) Bisects the chord.
(*c*) Is perpendicular to the chord.
(*d*) Bisects the arc.

6. A tangent to a circle is perpendicular to the radius drawn to the point of tangency (Fig. 88), and conversely.

7. Arcs included between parallel lines are equal.

Fig. 88.

8. An inscribed angle is measured by one-half the intercepted arc (Fig. 89).

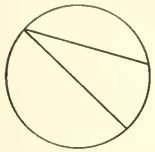


FIG. 89.

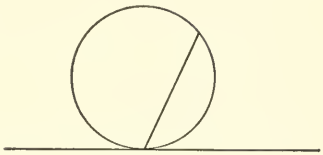


FIG. 90.

9. An angle formed by a tangent and a chord Fig. (90) is measured by one-half the intercepted arc.

10. An angle formed by two secants meeting outside a circle (Fig. 91) is measured by one-half the difference of the intercepted arcs.

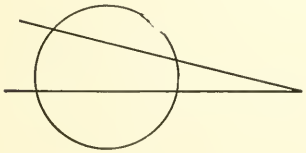


FIG. 91.

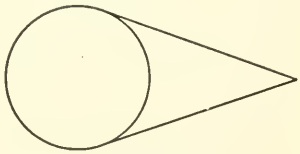


FIG. 92.

11. Two tangents drawn to a circle from an external point (Fig. 92) are equal.

110 Ratio and Proportion.

1. If two quantities are in proportion the product of the extremes equals the product of the means:

i.e., if $a : b :: c : d,$

then $ad = bc.$

2. A line parallel to one side of a triangle divides the other two sides proportionally.

3. The medians of a triangle meet in a point two-thirds of the distance from the vertex to the mid-point of the opposite side.

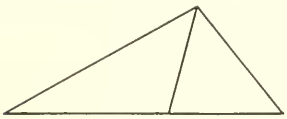


Fig. 93.

4. The bisector of the angle of a triangle (Fig. 93) divides the opposite side into segments proportional to the adjacent sides.

111. Similar Triangles. Triangles having their sides proportional and angles equal are **similar.**

1. Two triangles are similar if two angles of the one are equal to two angles of the other.

2. Two triangles are similar if the three sides of the one are proportional to the three sides of the other.

3. Triangles having their corresponding sides parallel or perpendicular are similar.

112. Right Triangles.

1. Two right triangles having an acute angle of the one equal to an acute angle of the other are similar.

2. The square on the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

3. The perpendicular from the vertex of the right angle of a right triangle to the hypotenuse divides the triangle into two triangles similar to each other and similar to the original triangle.

4. The perpendicular to the hypotenuse of a right triangle is the mean proportional between the segments of the hypotenuse.

5. Either side is the mean proportional between the

whole hypotenuse and the segment of the hypotenuse adjacent to that side.

EXERCISE 1

1. Find the other angles of the parallelogram, Fig. 94.

Ans. $B = 126°$.
$C = 54°$, $D = 126°$.

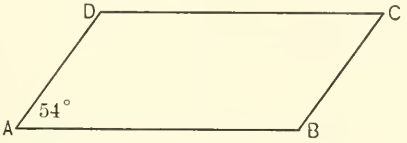


Fig. 94.

2. Find the value of the following angles in the regular inscribed hexagon Fig. 95:

(*a*) *BOC*.	Ans.	60°.
(*b*) *BEC*.		30°.
(*c*) *AFE*.		120°.
(*d*) *ECD*.		30°.

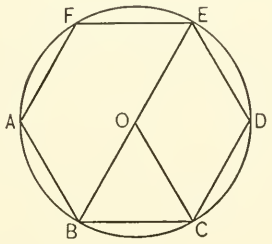


Fig. 95.

3. If a stick 10 ft. long casts a shadow 8 ft., how high is a tree that casts a shadow 55 ft.? Ans. 68.7 ft.

4. In the right triangles, Fig. 96, find x.

Ans. $x = 4.2896$.

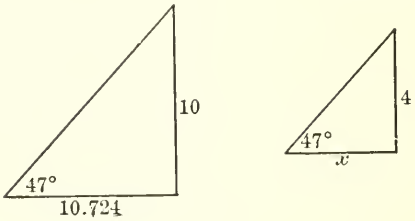


Fig. 96.

5. In the triangle, Fig. 97, find the hypotenuse.

Ans. 20 ft.

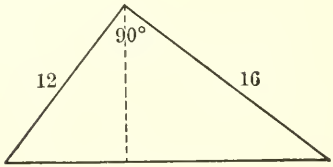


Fig. 97.

6. Find the segments of the hypotenuse.

Ans. $7\frac{1}{5}$, $12\frac{4}{5}$.

7. Find the perpendicular to the hypotenuse. Ans. $9\frac{3}{5}$.

8. Find the area. Ans. 96.

CHAPTER XII

LOGARITHMS

113. Aids to Computation. Various means have been devised for simplifying the work of long multiplication and division, such as necessarily occurs in the use of trigonometric functions and in other places. Three of these means are in common use:

I. Calculating Machines. They are always expensive and not easily carried about.

II. Slide Rules. Inexpensive instruments and easily carried, but not sufficiently accurate for some work.

III. Logarithms. Tables of logarithms are very nexpensive and easily carried about and give more accurate results than the slide rule.

Logarithms were first put into practical use by John Napier in 1614, later they were improved by Henry Briggs. Logarithms are used extensively in computations because they reduce multiplication, division and finding roots to easy, rapid calculations. Logarithms are also necessary in solving certain equations and are used in some formulas.

114. Common Logarithms. The common **logarithm** of a number is the exponent of 10 which makes the power of 10 equal to that number. For example, $10^2 = 100$, therefore 2 is the logarithm of 100.

10 is the base of the logarithm in this definition; other bases are used in higher work. The student must keep in mind the fundamental conception of a logarithm, namely, that "**a logarithm is an exponent.**"

115. Finding the Logarithm of a Number. By the definition of a logarithm,

$$10^2 = 100 \quad \text{therefore } \log 100 = 2,$$
$$10^3 = 1000 \text{ therefore } \log 1000 = 3.$$

Hence the logarithm of 254 must lie between 2 and 3. That is, it must be 2+; carried to five places it is 2.40483.

A logarithm is composed of two parts, a whole number and a decimal. The decimal part of the logarithm is the **mantissa** and can be found in the table of logarithms. The mantissa is always positive, and is independent of the position of the decimal point in the number. That is, the mantissa is determined by the figures only. For example, the mantissa of 3472 is the same as the mantissa of 34.72.

The whole number part of a logarithm is the **characteristic.** The characteristic cannot be found in the table, but must be determined by rule. The characteristic is sometimes positive and sometimes negative, as in the case of decimals. If the characteristic is -3 and the mantissa is .44632, the whole logarithm cannot be written -3.44632, as this would indicate that the mantissa also is negative. The mantissa is never negative and so the following notation is used, $\bar{3}.44632$, which means that the characteristic alone is negative.* The student should learn to distinguish between characteristic and mantissa.

116. Finding the Characteristic. The characteristic can be determined by definition thus:

$$10^2 = 100 \quad \text{therefore } \log 100 = 2,$$
$$10^3 = 1000 \quad \text{therefore } \log 1000 = 3,$$
$$10^4 = 10000 \text{ therefore } \log 10000 = 4.$$

Hence the characteristic of the logarithm of all numbers

* Note. The following notation is sometimes used for negative characteristics: $7.44632 - 10$.

between 100 and 1000 is 2 and of all numbers between 1000 and 10000 is 3, etc. If the number of figures in the whole number is increased or decreased by 1, the characteristic of the logarithm of the number is increased or decreased by 1. Hence:

For numbers between			The characteristic is
1000	and	10000	3
100	and	1000	2
10	and	100	1
1	and	10	0
.1	and	1	−1
.01	and	.1	−2
.001	and	.01	−3

This rule is too cumbersome for efficient use so the following rule will be used. Observe the following table of numbers and their logarithms:

Number.			Logarithm.
	units		
	0	.00254	$\bar{3}.40483$
	0	.0254	$\bar{2}.40483$
	0	.254	$\bar{1}.40483$
	2	.54	0.40483
2	5	.4	1.40483
25	4	.	2.40483
254	0	.	3.40483
2540	0	.	4.40483

Observe that the characteristics in the above table could be found by counting from the units figure (the figure in the column between the lines) to the 2.

Rule. *To find the characteristic count from (not including) the units figure to the first significant figure.*

Observe that:

1. *If the count is to the left, the characteristic is positive, and if to the right, the characteristic is negative.*

2. *A significant figure means a figure other than zero, and the first significant figure means the first figure (other than zero) on the left of a number.*

Example 1. What is the characteristic of 7934? Count from the 4 to the 7, thus $\overset{3\,2\,1}{7934}$. The characteristic is 3.

Example 2. What is the characteristic of 0.000853? Count from the first 0 to the 8, thus $0.\overset{1\,2\,3\,4}{0}00853$. The charcteristic is -4.

EXERCISE 1

What is the characteristic in the following numbers?

1. 25	6. 0.0623
2. 354	7. 0.00043
3. 3540	8. 0.0004
4. 75.84	9. 0.421
5. 66319	10. 6.3

117. Finding the Mantissa. The mantissa must be found in the table of logarithms. The first two figures of a number will be found in the column marked N at the left of the page and the third figure will be found at the top of the page. The mantissa will be found in the row with the first two figures and in the column of the third figure.

Example 1. Find the mantissa of 567. Look for 56 in the column at the left of the page and find the mantissa (.75358) in that row and in the column headed 7. The complete logarithm is 2.75358.

In finding a logarithm the following steps should be followed:

1. Write the number thus, log 567 =
2. Determine the characteristic, log 567 = 2.——
3. Find the mantissa, log 567 = 2.75358.

For efficiency in finding the mantissa, fix the entire mantissa in mind before writing any of it.

EXERCISE 2

Find the logarithms of the following:

1. 567.	7. 9.
2. 983.	8. 0.234.
3. 75.4.	9. 0.0056.
4. 6330.	10. 7.03.
5. 565.	11. 40.2.
6. 230.	12. 768.

118. Interpolation. The logarithm of 4683 cannot be found in an ordinary logarithm table, but the logarithms 4680 and 4690 can be found. The logarithm of 4683 must be determined by interpolation between these two logarithms. The work can be arranged as follows:

$$\begin{aligned} \log 4680 &= 3.67025 \\ \log 4690 &= 3.67117 \\ \hline & \quad 92 \text{ (Tabular difference)} \end{aligned}$$

Take .3 of this difference and add to the logarithm of 4680, thus $.3 \times 92 = 28$, that is, 28 in the last two places.

$$\begin{aligned} \log 4680 &= 3.67025 \\ &\quad +28 \\ \hline & 3.67053 = \log 4683 \end{aligned}$$

In general, if the fourth figure of a number is different than 3, as 7, take .7 of the tabular difference and add to the

logarithm of the smaller number. In interpolation do not carry the results beyond the fifth place.

The above method is too elaborate for rapid efficient use of logarithms, and for efficiency, interpolation must be done mentally. The following steps and suggestions should prove helpful:

Example 1. Find the logarithm of 753.4.

I. Write the characteristic.

II. Look for the mantissas of 753 and 754; they will be found side by side, thus: 87679 and 87737.

III. Subtract mentally, getting the difference of 58 (in the last two places).

IV. $.4 \times 58 = 23.2$ or 23 in the last two places.

V. $87679 + 23 = 87702$.

VI. Write the mantissa 87702 after the characteristic.

NOTE. Tables in this book will give results correct to three or four figures. Students desiring more accurate results can obtain more complete tables in a small compact book at a moderate price.

EXERCISE 3

Find the logarithms of the following:

1. 6346.	Ans. 3.80250.
2. 8437.	3.92619.
3. 245.2.	2.38952.
4. 3.657.	0.56312.
5. 5637.	$\bar{1}$.75105.

119. Multiplication by Logarithms. Recall the fundamental part of the definition of a logarithm, namely, that "A logarithm is an exponent," and recall the law of exponents for multiplication whereby $x^3 \times x^4 = x^7$. It will be seen that the sum of the logarithms of two numbers is the

logarithm of their product. Therefore the following rule can be stated:

Rule. *To multiply two numbers add their logarithms and find the number corresponding to the sum.*

120. Antilogarithms. The number which corresponds to a given logarithm is called its **antilogarithm.**

Example 1. Find the antilogarithm of 2.94576. Look for the mantissa (94596) in the main part of the table. The first two figures of the number (88) will be found in the column marked N at the left side of the page, in the row in which 94596 is found. The third figure (3) is found at the top of the column in which 94596 is found. Therefore the figures corresponding to the mantissa 94596 are 883. The position of the decimal point is determined by the characteristic. The method is the opposite of the method of finding the characteristic. For example, in the above problem the characteristic is 2 and the figures are 883. Therefore, count 2 from the 8 thus:

$$\overset{12}{883}$$

The next figure is units thus:

$$\overset{12\ \text{units}}{883}$$

Rule. *Count, from the first figure of the number obtained, as many places as indicated by the characteristic, the next figure will be units. If the characteristic is positive, count to the right, and if the characteristic is negative, count to the left. Zeros will have to be supplied in this case.*

If the mantissa cannot be found exactly in the tables, interpolate between the two numbers corresponding to the two nearest logarithms.

Example. Find the antilog of $\bar{3}.56505$.

$$56467 = \log 367$$
$$56585 = \log 368$$
(Tabular difference) $\quad 118$
$$56467 = \log 367$$
$$56505 = \log \text{ required number}$$

Difference $\quad 38$

$$\frac{\text{Diff.}}{\text{Tab. diff.}} = \frac{38}{118} = .3+$$

Therefore $56506 = \log 3673\frac{38}{118} = \log 3673+$

The figures of the antilogarithm are 3673, the characteristic is $\bar{3}$, therefore count as follows:

units
3 2 1
0.003673. Ans.

In interpolation results are not carried more than one place beyond the figures given in the table.

EXERCISE 4

Find the antilogarithms of the following logarithms:

1. 2.58286.	Ans. 382.7.
2. 4.59106.	39000.
3. 0.76384.	5.8055.
4. $\bar{2}$.84365.	.069767.
5. 1.08458.	12.15.
6. $\bar{1}$.23615.	.17225.
7. 0.29336.	1.965.
8. 5.25527.	180000.

121. Multiplication. The work of multiplication by logarithms should be arranged as follows:

Multiply 3675 by 78.56.

$$\begin{aligned} \log\ 3675 &= 3.56526 \\ \log\ 78.56 &= 1.89520 \\ \hline & 5.46046 \text{ (Adding)} \\ \text{Ans.} & \quad 288710. \text{ (Antilog)} \end{aligned}$$

EXERCISE 5

Perform the following multiplications by means of logarithms:

1. 243×375.	Ans. 91126.
2. 3984×5.6	22310.
3. $.024 \times 7685$.	184.44.
4. $.654 \times .4379$.	.28639.
5. 76×5.	380.
6. 24765×3.49.	86432.
7. $.0024 \times 76587$.	183.81.
8. $.653 \times 378 \times 55.15$.	13613.
9. $273 \times 68.49 \times 23.69$.	442960.
10. $.002 \times .0347 \times 56$.	.0038865.
11. $127000 \times .5634$.	71552.
12. 653.14×5.7823.	3776.7.

122. Division. Since

$$x^5 \div x^2 = x^3,$$

and since a logarithm is an exponent, the logarithm of the quotient of two numbers is the *difference* of the logarithms of the numbers. The logarithm of the divisor is to be subtracted from the logarithm of the dividend. The antilogarithm of the difference is the quotient.

123. Cologarithms. The above method of division is too inefficient when several numbers are to be multiplied and divided. For example:

$$\frac{254\times781}{53\times125}$$

Add:
$$\log 254 = 2.40483$$
$$\log 781 = 2.89265$$
$$5.29748$$

Add:
$$\log\ 53 = 1.72428$$
$$\log 125 = 2.09691$$
$$3.82119$$

Subtract:
$$5.29748$$
$$3.82119$$
$$1.47629 = \log 29.943$$

Therefore: $\dfrac{254\times781}{53\times125} = 29.943.$

But $\dfrac{254\times781}{53\times125}$ can be written $254\times781\times\frac{1}{53}\times\frac{1}{125}$, and, if the logarithms of $\frac{1}{53}$, and $\frac{1}{125}$ can be found readily, this problem reduces to multiplication, and hence only one addition is necessary: $\frac{1}{53} = 1 \div 53.$

$$\log \tfrac{1}{53} = \log 1 - \log 53$$
$$\log \tfrac{1}{53} = 0 \quad - \log 53 \quad (\log 1 = 0)$$
$$\log \tfrac{1}{53} = -\log 53$$

The work can be arranged as follows:

Subtract:
$$0$$
$$\log 53 = 1.72428$$

Borrow 1 from 0 leaving -1, thus:

$$\begin{array}{r} \bar{1}.9999^{10} \\ 1.72428 \\ \hline \bar{2}.27572 \end{array}$$

The logarithm of $\frac{1}{\text{a number}}$ is the **cologarithm** of the number. The above problem now reduces to:

Add:

$$\begin{array}{rl} \log 254 = & 2.40483 \\ \log 781 = & 2.89265 \\ \operatorname{colog}\ 53 = & \bar{2}.27572 \\ \operatorname{colog}\ 125 = & \bar{3}.90309 \\ \hline & 1.47629 = \log 29.943. \text{ Ans.} \end{array}$$

For efficiency the mantissa of the cologarithm must be obtained mentally, from the tables. The work can be done as follows: Find the mantissa of the logarithm, for example 34624, subtract each of the first four figures from 9 and the last one from 10, fix these numbers in mind and write all down at once thus, 65376. For the characteristic remember the $\bar{1}$ in the minuend and the laws for algebraic subtraction. As the student becomes more familiar with the work the characteristic of the cologarithm can be found as readily as the characteristic of the logarithm.

Rule. *To divide numbers by means of logarithms add the logarithms of all the dividends and the cologarithms of all the divisors and find the antilogarithm of the sum.*

The work should be arranged as follows;

Example. $\dfrac{246 \times 67.85}{23.5 \times .04732}$

Solution.

$$\begin{aligned} \log 246 &= 2.39094 \\ \log 67.85 &= 1.83155 \\ \operatorname{colog} 23.5 &= \bar{2}.62893 \\ \operatorname{colog} .04732 &= 1.32496 \\ & \hline \\ & 4.17638 \end{aligned}$$

Ans. 15010.

EXERCISE 6

Solve the following by logarithms:

1. $\dfrac{234\times451}{635\times7}$. Ans. 23.743.
2. $\dfrac{49.82\times6.731}{5.347\times21}$. 2.9864.
3. $\dfrac{6542\times31.67}{0.427\times167}$. 29.054.
4. $\dfrac{.0256\times68.715}{762\times45.19}$. .000051085.
5. $\dfrac{3.1416\times258.9}{275\times.02}$. 147.88.
6. $\dfrac{6543\times289}{21.32\times1.318}$. 67293.
7. $\dfrac{35\times24.32}{65\times22}$. 0.59525.
8. $\dfrac{165\times6941}{23.4\times4.675\times46.75}$. 223.94.

124. Powers of Numbers:

$$5^2=5\times5=25$$

Therefore,

$$\log 5^2=\log 5+\log 5=2\log 5$$

Similarly,

$$\log 5^3 = \log 5 + \log 5 + \log 5 = 3 \log 5$$

From which the following rule can be stated:

Rule. *To raise a number to a power multiply the logarithm of the number by the exponent of the number and find the antilogarithm.*

Example. Find 37^3.

$$\log 37 = 1.56820$$
$$3$$
$$4.70460$$
$$37^3 = 50652$$

EXERCISE 7

Find by means of logarithms:

1. 6^5.	Ans. 7776.
2. $.37^4$.	.018741.
3. 5.37^3.	154.85.
4. 1.257^3.	1.9862.
5. $.1254^3$.	0.001972.

125. Roots of Numbers.

$$25 = 5^2$$

$$\log 25 = 2 \log 5$$

$$\sqrt{25} = 5$$

$$\log \sqrt{25} = \log 5$$

$$2 \log \sqrt{25} = 2 \log 5$$

But

$$\log 25 = 2 \log 5$$

Therefore

$$2 \log \sqrt{25} = \log 25$$

$$\log \sqrt{25} = \tfrac{1}{2} \log 25$$

Similarly,

$$\log \sqrt[3]{64} = \tfrac{1}{3} \log 64$$

From the above the following rule can be stated:

Rule. *To find the square root of a number divide the logarithm of the number by 2 and find the antilogarithm.*

To find the cube root of a number divide the logarithm by 3 and find the antilogarithm.

To find any root of a number divide the logarithm of the number by the index of the root and find the antilogarithm.

NOTE. The index of a root is the small number placed above and to the left of the radical sign, and indicates which root is to be taken. For example, in $\sqrt[3]{75}$, 3 is the index of the root.

Example 1. Find $\sqrt[3]{75}$.

$$\log 75 = 1.87506$$
$$\div 3 = \ .62502$$
$$4.2172 = \sqrt[3]{75}$$

Example 2. Find $\sqrt[3]{.05}$.

$$\log .05 = \bar{2}.69897$$

$\bar{2}.69897$ cannot be divided by 3 as in the previous problem, since 3 will not go in $\bar{2}$ evenly, and a negative remainder cannot be carried over into the positive mantissa. But

$\bar{2}.69897$ can be written

$$-2 + \ .69897,$$

or

$$-3+1.69897.$$

Then

$$3 \mid \begin{array}{l} -3+1.69897 \\ \hline -1+\ .56632 \end{array}$$

or

$$\bar{1}.56632$$

hence

$$\sqrt[3]{.05} = .3684$$

EXERCISE 8

Find the following:

1. $\sqrt[3]{50}$.	Ans.	3.684.
2. $\sqrt[3]{137.5}$.		5.1614.
3. $\sqrt{.3}$.		.54773.
4. $\sqrt{8700}$.		93.274.
5. $\sqrt[3]{\left(\frac{34.28\times 57.5}{48.08\times 5.962}\right)^2}$.		3.6161.

126. Equations Containing an Unknown Exponent.

Axiom. If two quantities are equal, their logarithms are equal. Therefore if

$$3^x = 81$$

$$\log 3^x = \log 81$$

$$x \log 3 = \log 81$$

$$x \times .47712 = 1.90849$$

$$x = \frac{1.90849}{.47712}$$

$$x = 4$$

EXERCISE 9

Solve for x:

1. $5^x = 625$.	Ans. 4.
2. $3^x = 243$.	5.
3. $2^x = 19$.	4.2479.
4. $4^{x+1} = 256$.	3.

EXERCISE 10

Miscellaneous Problems

1. Given $V = \frac{4}{3}\pi r^3$ as the formula for the volume of a sphere, where r is the radius. Find the volume of a sphere having a radius of 13.34 in. Ans. 9944.3.

2. Find the diameter of a steel ball weighing 16 lbs. if steel weighs .283 lb. per cubic inch. Ans. 4.762.

3. Evalute the formula

$$C = \frac{.00388.}{\log \frac{s}{r}} \quad \text{(Capacity of aerial wire)}$$

when $s = 24$ and $r = \frac{1}{4}$. Ans. .0019574.

4. The formula for compound interest, compounded annually, is $A = a(1+r)^n$, where A = the amount at the end of n years, a is the original investment, r is the rate, and n the number of years. Find the amount of \$150.00 at the end of 20 years, interest compounded annually at 4 per cent. Ans. \$328.62.

5. How many years will it take \$100.00 to amount to \$240.66? Interest compounded annually at 5 per cent.

Ans. 18 years.

6. Evaluate the formula,

$$L = .0017 \times l \times \log \frac{d}{R}$$ (Inductance in a transmission line)

when $l = 25$, $d = 18$, and $R = .25$. Ans. .07894.

7. Evaluate the formula,

$$C = \frac{2.413\ Kl}{10^7 \log \frac{D}{d}}$$ (Capacity of a submarine cable)

when $K = 6$, $l = 1000$, $D = 2.5$, $d = 1.5$. Ans. 0.00666.

8. Evaluate the formula for the capacity of a submarine cable when $C = .25$, $l = 35{,}000$, $D = 3$, $d = 1.75$.

Ans. 6.929.

CHAPTER XIII

RIGHT TRIANGULATION

127. Constant Ratios in Right Triangles. The right triangles shown in Fig. 98 are similar since they have an

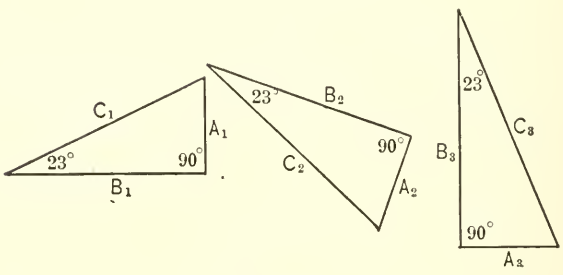


Fig. 98.

acute angle of the one equal to an acute angle of the other.

$$\frac{a_1}{c_1}=\frac{a_2}{c_2}=\frac{a_3}{c_3} \quad \text{(why?)}$$

Therefore all right triangles having one acute angle equal to 23° are similar, and therefore the ratio of the side opposite 23° to the hypotenuse in each triangle is constant (always the same). By actual measurement of the sides this ratio reduced to a decimal correct to .00001 can be calculated to be 0.39073.

In the same manner:

$$\frac{b_1}{c_1}=\frac{b_2}{c_2}=\frac{b_3}{c_3}=0.92050$$

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}=0.42447$$

$$\frac{b_1}{a_1}=\frac{b_2}{a_2}=\frac{b_3}{a_3}=2.35585$$

$$\frac{c_1}{b_1}=\frac{c_2}{b_2}=\frac{c_3}{b_3}=1.0864$$

$$\frac{c_1}{a_1}=\frac{c_2}{a_2}=\frac{c_3}{a_3}=2.5593$$

Example 1. In the right triangle of Fig. 99,

$$\frac{a}{60}=0.39073$$

$$a=23.4438$$

$$\frac{b}{60}=0.9205$$

$$b=55.230$$

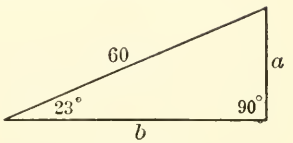


Fig. 99.

Example 2. In the right triangle of Fig. 100,

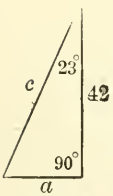


Fig. 100.

$$\frac{a}{42}=0.42477$$

$$a=?$$

$$\frac{c}{42}=1.0864$$

$$c=?$$

Observe that: *a is the side* **opposite** 23°, *b is the side* **adjacent** *to* 23°, *and c is he* **hypo enuse** *of he right triangle.*

For the purpose of tabulating these **constant ratios** it is convenient to refer to them by names Referring to Fig. 101,

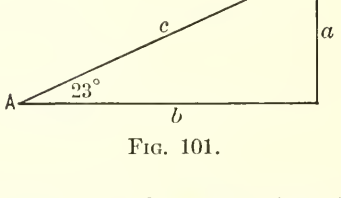


Fig. 101.

$\frac{a}{c}$ is called the **sine** of 23°, written sin 23°.

$\frac{b}{c}$ is called the **cosine** of 23°, written cos 23°.

$\frac{a}{b}$ is called the **tangent** of 23°, written tan 23°.

$\frac{b}{a}$ is called the **cotangent** of 23°, written cot 23°.

$\frac{c}{b}$ is called the **secant** of 23°, written sec 23°.

$\frac{c}{a}$ is called the **cosecant** of 23°, written csc 23°.

	sin	cos	tan	cot	sec	csc
23°	0.39073	0.92050	0.42447	2.3559	1.0864	2.5593

These ratios can be calculated for any angle and are called functions of the angle. In general, for any value of angle A,

$$\sin A = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos A = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan A = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

$$\csc A = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}}$$

By using a table of these ratios for all angles, if one side and one acute angle of a right triangle are known all the remaining parts can be calculated. Table IV contains these ratios for all angles.

128. Use of the Tables.

Example 1. Find the sine of 37° 20′. Locate the sine page (Table IV). Find 37° in the column at the left. The sine of 37° 20′ is found in the row marked 37° and in the column marked 20′ at the top.

Example 2. Find the cosine of 52° 50′. Find the cosine page (Table IV). (Cosine is written at the bottom of the page.) Find the degrees in the column at the right and the minutes at the bottom of the page.

Observe that: To find the sin or tan, locate the angle in degrees at the left of the page and the minutes in the column at the top. To find the cos or cot, locate the angle in degrees at the right of the page and the minutes in the column at the bottom.

Example 3. Find a and b in Fig. 102.

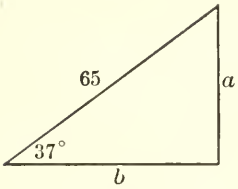


FIG. 102.

$$\frac{a}{65} = \sin 37°.$$

$$\frac{a}{65} = .60181.$$

$$a = 39.1176.$$

To find b:

$$\frac{b}{65} = \cos 37°.$$

$$\frac{b}{65} = .79863.$$

$$b = 51.9109$$

To find $\angle B$:

$$\angle A + \angle B = 90°.$$

$$37° + \angle B = 90°.$$

$$\angle B = 53°.$$

Check.

$$c^2 = a^2 + b^2$$

$$65^2 = 39.1176^2 + 51.9109^2$$

$$4225 = 1530.2 + 2694.8 +$$

$$4225 = 4225 +$$

NOTE. Tables correct to five figures will give results correct to five figures.

Example 4. Find b, c, and $\angle A$ in the triangle of Fig. 103.

To find $\angle A$:

$$\angle A + \angle B = 90°.$$

$$\angle A + \angle 59° = 90°.$$

$$\angle A = 31°.$$

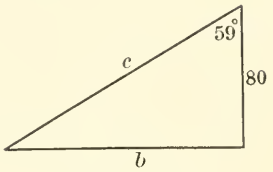


FIG. 103.

To find b:

$$\frac{b}{80} = \cot 31°.$$

$$\frac{b}{80} = 1.6643.$$

$$b = 133.144.$$

To find c:

$$\frac{80}{c} = \sin 31°.$$

$$\frac{80}{c} = .51504.$$

$$80 = .51504c$$

$$c = 155.328.$$

Observe: To find b, $\frac{a}{b} = \tan A$ might have been used, which would give

$$\frac{80}{b} = .60090.$$

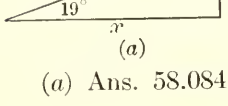
$$b = \frac{80}{.60090}.$$

$$b = 133.1+$$

By selecting the ratio which has the unknown side for numerator, the result is obtained by multiplying by a decimal rather than by dividing by a decimal.

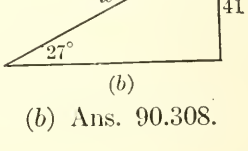
EXERCISE 1

Find x in the triangles of Fig. 104:



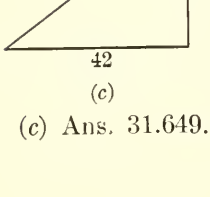
(*a*)

(*a*) Ans. 58.084.



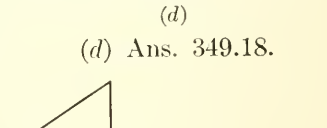
(*b*)

(*b*) Ans. 90.308.



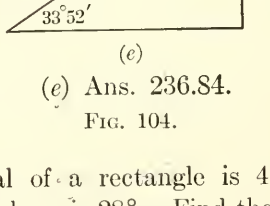
(*c*)

(*c*) Ans. 31.649.



(*d*)

(*d*) Ans. 349.18.



(*e*)

(*e*) Ans. 236.84.

FIG. 104.

2. The diagonal of a rectangle is 48″ and the angle between it and the base is 28°. Find the dimensions of the rectangle. Ans. 22.535; 42.361.

3. The altitude of an isosceles triangle is 15″ and the base angles are 37° 5′. Find the sides of the triangle.

Ans. 24.876; 24.876; 39.690.

4.

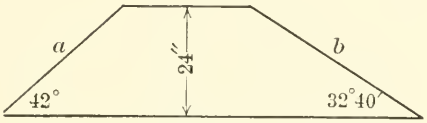


Fig. 105.

Find a and b in the trapezoid of Fig. 105.

Ans. $a=35.868$; $b=44.465$.

5. The altitude of an isosceles triangle is 24″ and the angle at the vertex is 97° 28′. Find the sides of the triangle.

Ans. 36.386; 36.386; 54.701.

6. The angle between the two sides of a parallelogram is 43° 15′ and one of the sides is 3.5″ long. Find the distance between the other two parallel sides. Ans. 2.3981.

129. Solution of a Right Triangle, Two Sides Given. If two sides of a right triangle are known, all the remaining parts can be calculated. The third side can be found by the formula $c^2=a^2+b^2$ and the angles can be found from Table IV.

Example 1. Find b, $\angle A$ and $\angle B$ in the right triangle of Fig. 106.

To find $\angle A$:

$$\frac{203.37}{500}=\sin A.$$

$$.40674=\sin A.$$

$$.40674=\sin 24°.$$

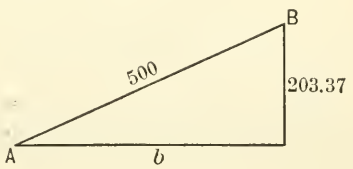


Fig. 106.

$$\angle A = 24°.$$

$$\angle B = 90° - 24° = 66°.$$

To find b:

$$\frac{b}{500} = \cos 24°.$$

$$\frac{b}{500} = .91355.$$

$$b = 456.775.$$

NOTE. b could have been found by using the formula $c^2 = a^2 + b^2$.

Example 2. Find $\angle A$ in the right triangle of Fig. 107.

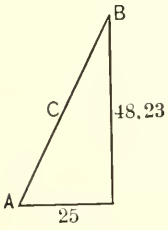


FIG. 107.

$$\frac{a}{b} = \tan A.$$

$$\frac{48.23}{25} = \tan A.$$

$$1.9292 = \tan A.$$

$$1.9292 = \tan 62°\ 36'.$$

$$\angle A = 62°\ 36'.$$

Example 3. Find $\angle B$ in the triangle of Fig. 108.

$$\frac{40}{70.13} = \cos B$$

$$.57036 = \cos B.$$

$$\angle B = 55°\ 13'.$$

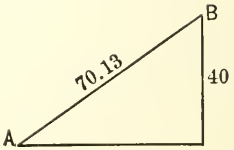


FIG. 108.

NOTE. When the decimal does not exactly correspond to one the table, the required angle can be expressed in degrees, minutes a seconds. For the present, angles will be expressed to the ne minute.

EXERCISE 2

1. Find $\angle A$, Fig. 109.

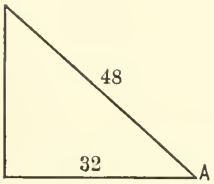


Fig. 109.

Ans. 48° 11′.

2. Find $\angle x$, Fig. 110.

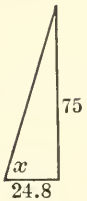


Fig. 110.

Ans. 71° 42′.

3. Find x, Fig. 111.

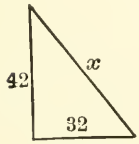


Fig. 111.

Ans. 52.802.

4. Find x, Fig. 112.

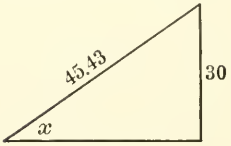


Fig. 112.

Ans. 41° 20′.

5. The base of an isosceles triangle is 21″ and the equal sides are each 16″. Find the base angles. Ans. 48° 59′.

6. The base of an isosceles triangle is 48″ and the altitude is 32″. Find the vertex angle. Ans. 73° 44′.

7. The dimensions of a rectangle are 78″ and 64″. Find the angle included between the diagonal and the longer side. Ans. 39° 22′.

8. One side of a parallelogram is 2.25″ and the distance between the other two parallel sides is 1.75″. Find the angles of the parallelogram. Ans. 51° 3′.

9. The diagonals of a rhombus are 58″ and 76″. Find the angles of the rhombus. Ans. 74° 42′; 105° 18′.

10. The non-parallel sides of a trapezoid are 48″ and 54″ and the altitude is 36″. Find the angles of the trapezoid. Ans. 48° 35′, 131° 25′; 41° 49′, 138° 11′.

130. Interpolation. Most tables of the functions of angles give the values for degrees and minutes only, or to every ten minutes. When a function of an angle involving seconds is to be used it must be determined from the functions of the two nearest angles given.

Example 1. Find sin 27° 27′.

From the tables,

$$\sin 27^\circ\ 30' = .46175$$
$$\sin 27^\circ\ 20' = .45917$$

Difference for 10′ $= .00258$ (tabular difference)

Difference for 7′ is $\frac{7}{10}$ of $.00258 = .00181$.

Adding this difference to $\sin 27^\circ\ 20' = .45917$

gives $.46098 = \sin 27^\circ\ 27'$

Example 2. Find tan 44° 16′.

$$\tan 44^\circ\ 20' = .97700$$
$$\tan 44^\circ\ 10' = .97133$$

567 (Tabular difference)

$\frac{6}{10}$ of $567 = 340$ (in the last 3 places)

$$\tan 44^\circ\ 10' = .97133$$
$$+\ 340$$
$$= .97473 = \tan 44^\circ\ 16'$$

Example 3. Find cos 17° 22′.

$$\begin{aligned}
&\cos 17^\circ\ 20' = .95459\\
&\cos 17^\circ\ 30' = .95372\\
&\qquad\qquad\qquad\quad 87 \quad \text{(Tabular difference)}\\
&\tfrac{2}{10} \text{ of } 87 = 17\\
&\cos 17^\circ\ 20' = .95459\\
&\qquad\qquad\quad -17\\
&\qquad\qquad .95442 = \cos 17^\circ\ 22'
\end{aligned}$$

Observe that: 1. *The cosine and the cotangent of an angle vary inversely as the angle.*

2. *In interpolation for the cosine (or cotangent) the fractional part of the tabular difference must be subtracted from the cosine (or cotangent) of the smaller angle.*

3. *When a five-place table is used in interpolation the result is never carried beyond the fifth place.*

For efficiency in interpolation the tabular difference must be found mentally. The fractional part of the tabular difference can generally be determined mentally. This difference should be added to (or subtracted from) the function of the smaller angle mentally.

EXERCISE 3

Find

1. sin 27° 15′.	Ans. .45787.
2. sin 65° 29′.	.90984.
3. tan 32° 41′.	.64158.
4. cos 43° 12′.	.72897.
5. tan 28° 32′.	.54371.
6. cot 6° 32′.	8.73262.
7. tan 85° 17′.	12.1201.
8. cos 64° 18′.	.43365.

131. Finding the Angle that Corresponds to a Given Function.

Example 1. Find $\angle x$ if $\tan x = .63707$.

Find 63707 on the tangent page of Table IV. It will be found to lie in the 32° row and 30′ column. Therefore

$$\angle x = 32^\circ\ 30'$$

Example 2. If $\sin x = .23137$, find $\angle x$.

Look for .23137 on the sin page of Table IV. It cannot be found, but

$$.23062 = \sin 13^\circ\ 20'$$
$$.23345 = \sin 13^\circ\ 30'$$

Tabular diff. = 283

Therefore x lies between 13° 20′ and 13° 30′:

$$23062 = \sin 13^\circ\ 20'$$
$$23137 = \sin x$$

Difference = 75

Therefore $x = 13^\circ\ 20' + \frac{75}{283}$ of $10' = 13^\circ\ 23'$.

In practice the work should be done mentally.

EXERCISE 4

Find x if:

1.	$\sin x = .63910$.	Ans. 39° 44′.
2.	$\sin x = .76912$.	50° 17′.
3.	$\cos x = .80653$.	36° 15′.
4.	$\cos x = .43347$.	64° 19′.
5.	$\cot x = 6.0776$.	9° 20′.
6.	$\tan x = 5.3977$.	79° 30′.

132. Logarithms of the Functions of Angles. Table III gives directly the logarithms of the trigonometric functions

of angles. The table is used the same as Table IV. The characteristic of the logarithm will be found in the table. Table III makes it possible to multiply and divide by trigonometric functions by means of logarithms without the use of two tables. It is recommended that logarithms be used in the remainders of the problems wherever possible.

133. Angle of Elevation and Depression. When an object is sighted above the horizontal plane, the angle which a line from the eye to the object makes with the horizontal plane is the *angle of elevation*, (Fig. 113). When an object is sighted below the horizontal plane, the angle which a line from the eye to the object makes with the horizontal plane is the *angle of depression*.

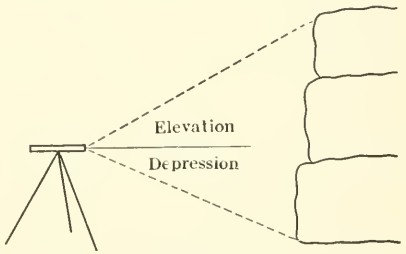


FIG. 113.

EXERCISE 5

1. Find the angle of elevation of the sun when a pole 20′ high casts a shadow 14′ 9″ long. Ans. 53° 35′ 30″.

2. A building 145′ high forms an angle of elevation of 18° 3′ from a point on level ground. How far is the observer from the building? Ans. 444.95.

3. The distance from the foot to the top of the hill is 220 yds. Find the height of the hill if it is of uniform grade of 7° 2′. Ans. 26.94.

4. The pitch of a roof is 40°, the width of the building is 28 ft. What length rafter must be used if they project 18 in. beyond the sides? Ans. 19.77.

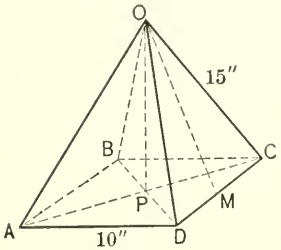


Fig. 114.

5. Find the height of a tree if it makes an angle of 25° 35′ at a point 25 ft. from the base of the tree. Ans. 11.96.

6. Find the base angle and the altitude of a cone if the slant height is 8 in. and the radius of the base is 6 in.

Ans. 41° 24′ 34″; 5.2915.

7. *ABCD–O*, Fig. 114, is a square pyramid having an edge of 15 in. and one side of the base 10″.

(*a*)	Find the angle *OCM*.	Ans. 70° 32′.
(*b*)	Find the slant height *OM*.	14.142.
(*c*)	Find the angle *OMP*.	90°.
(*d*)	Find the angle *OCP*.	61° 52′ 26″.
(*e*)	Find the altitude *OP*.	13.299.

8. Find *BC*, Fig. 115.

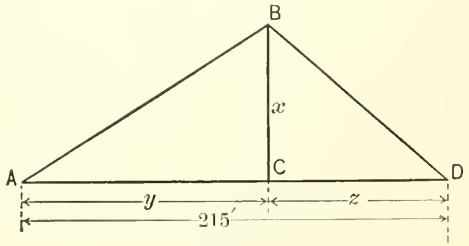


Fig. 115.

Let $y = AC$

and $z = CD$

and $x = CB$

$$y + z = 215$$

$$\frac{y}{x} = \cot 32^\circ\ 10'$$

$$y = 1.5900x$$

$$\frac{z}{x} = \cot 41^\circ\ 22'$$

$$z = 1.1356x$$

$$1.5900x + 1.1356x = 215$$

$$2.7256x = 215$$

$$x = 78.88$$

9. Two persons at A and B, Fig. 116, observe an airplane directly over the line between them. If the two observers are 2500 ft. apart and the angles of elevation are 65° 20′ and 21° 32′, find the height of the airplane. Ans. 835.12.

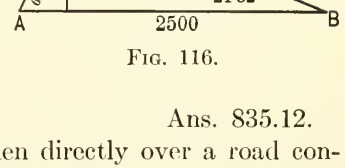


Fig. 116.

10. A man in a balloon when directly over a road connecting two towns 7 miles apart observes the angles of depression as in Fig. 117. Find the height of the balloon. Ans. 2.0361.

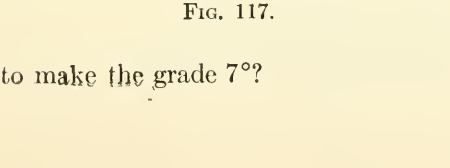


Fig. 117.

11. A hill has a grade of 9°. If the height of the hill AB is 59 ft., how much must be taken from the top of the hill to make the grade 7°? Ans. 13.26 ft.

CHAPTER XIV

TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

134. Functions as Lines. In the circle, Fig. 118, let the radius equal 1. Then

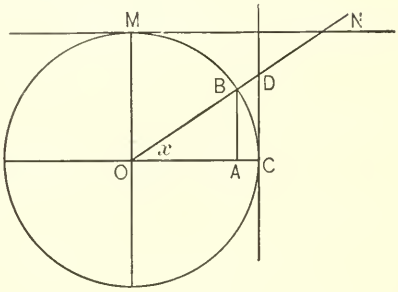


FIG. 118.

$$\sin x = \frac{AB}{OB} = \frac{AB}{1} = AB$$

$$\cos x = \frac{AO}{OB} = AO$$

$$\tan x = \frac{CD}{OC} = CD$$

$$\cot x \ (\angle x = \angle N) = \frac{MN}{MO} = MN$$

Then, if the line OC remains fixed and the line OB revolves about O, the angle x increases, but $\sin x$ is always the perpendicular distance from B to the line OC.

Thus, AB, $A'B'$, $A''B''$ in Fig. 119, represent the sine of the different values of x.

From Fig. 118 it can be seen that, as the line OB revolves and x changes from 0° to 90°,

sin x changes from 0 to 1,
cos x changes from 1 to 0,
tan x changes from 0 to ∞ (∞ = infinite value),
cot x changes from ∞ to 0.

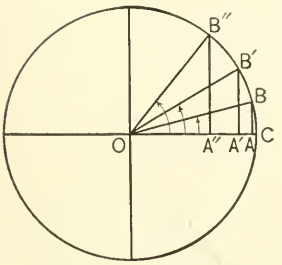


Fig. 119.

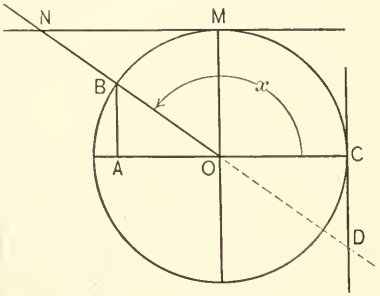


Fig. 120.

135. Value of the Functions of Angles Greater than 90°. When x becomes greater than 90°, Fig. 118 becomes Fig. 120. Here sine x cannot be defined as the $\frac{\text{adjacent side}}{\text{hypotenuse}}$, but is defined, on a broader meaning than before, as the perpendicular distance from B to the line CO, divided by the radius of the circle (= 1). That is,

AB represents sin x
OA represents cos x
CD represents tan x
MN represents cot x

136. Signs of the Functions of Angles Greater than 90°. As x increases from 0° to 90°, cos x decreases from 1 to 0°; but as x keeps on increasing from 90° to 180°, the cosine keeps changing in the same manner; that is, it decreases from 0 to -1, etc. Hence, in Fig. 124, the diameters must be taken as coordinate axes, where distances to the right are positive, and to the left negative, up positive and down negative.

From Fig. 121 it can be seen that, if x lies between 90° and 180°:

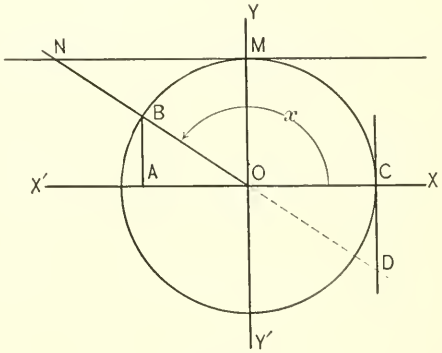


Fig. 121.

sin x $(=AB)$ is positive
cos x $(=AO)$ is negative
tan x $(=CD)$ is negative
cot x $(=MN)$ is negative

If x lies between 180° and 270°, as in Fig. 122:

sin x $(=AB)$ is negative
cos x $(=AO)$ is negative
tan x $(=CD)$ is positive
cot x $(=MN)$ is positive

The signs of the functions of the angles between 270° and 360° can be worked out from a similar figure. Functions of angles greater than 180° are very seldom used.

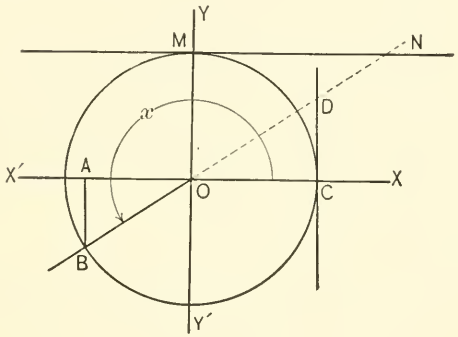


Fig. 122.

137. Values of Functions of Angles between 90° and 360°. In Fig. 123 triangles OAB, $OA'B$, $OA'B''$, and OAB''' are congruent (Geometry).

And $AB = A'B' = A'B'' = AB'''$ (corresponding parts). That is, for

Angles between 90° and 180°:

$$\sin\ (180° - x) = \sin x$$
$$\cos\ (180° - x) = -\cos x$$
$$\tan\ (180° - x) = -\tan x$$
$$\cot\ (180° - x) = -\cot x$$

Angles between 180° and 270°:

$$\sin\ (180° + x) = -\sin x$$
$$\cos\ (180° + x) = -\cos x$$
$$\tan\ (180° + x) = \tan x$$
$$\cot\ (180° + x) = \cot x$$

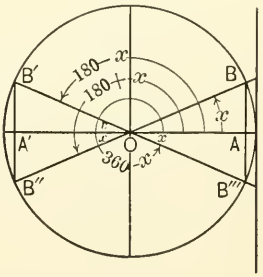


Fig. 123.

Angles between 270° and 360°:

$$\sin\ (360°-x) = -\sin\ x$$
$$\cos\ (360°-x) = \cos\ x$$
$$\tan\ (360°-x) = -\tan\ x$$
$$\cot\ (360°-x) = -\cot\ x$$

The values of the functions of angles greater than 90° can also be computed by drawing a similar figure and finding $\sin\ (90°+x)$ in terms of x, etc. This solution will be found in texts which are more complete in the theoretical part of trigonometry.

Example 1. Find sin 123°.

$$\sin 123° = \sin\ (180°-123°) = \sin 57° = .83867.$$

EXERCISE 1

Find:

1.	sin 100°.	Ans. .98481.
2.	sin 170°.	.17365.
3.	cos 160°.	−.93969.
4.	tan 150°.	−.57735.
5.	cot 120°.	−.57735.
6.	sin 200°.	−.34202.
7.	cot 240°.	.57735.
8.	cos 300°.	.5.
9.	tan 122° 45′.	−1.5547.
10.	sin 145° 32′.	.56593.
11.	cos 104° 7′.	−.24390.
12.	cot 125° 38′.	−.71681.

CHAPTER XV

OBLIQUE TRIANGLE

138. Law of Sines. In the triangle shown in Fig. 124, h is perpendicular to AB. Then,

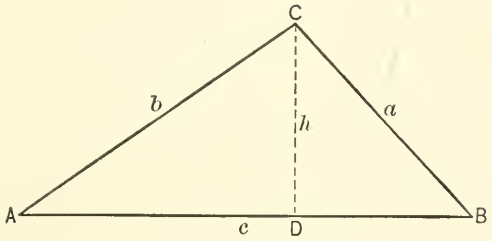


FIG. 124.

$$\sin A = \frac{h}{b}. \quad \ldots\ldots\ldots \quad (1)$$

$$\sin B = \frac{h}{a}. \quad \ldots\ldots\ldots \quad (2)$$

$$\frac{\sin A}{\sin B} = \frac{a}{b}. \quad \text{Dividing (1) by (2).}$$

Similarly it can be shown that:

$$\frac{\sin A}{\sin C} = \frac{a}{c}$$

and

$$\frac{\sin B}{\sin C} = \frac{b}{c}$$

By rearranging terms in the above equations they can be put in the form,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

The above equations are known as the **law of sines** and should be memorized.

Example 1. Solve the triangle of Fig. 125 for $\angle C$, and for x.

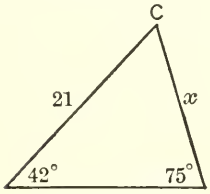


Fig. 125.

$$\angle C+42°+\ 75°=180°$$

$$\angle C+117°=180°$$

$$\angle C=\ 63°$$

$$\frac{x}{\sin 42°}=\frac{21}{\sin 75°}$$

$$x=\frac{21\times\sin 42°}{\sin 75°}$$

(The computation can be done most efficiently by means of logarithms)

$$x=\frac{21\times.66913}{.96592}$$

$$x=14.54$$

Example 2. Solve the triangle of Fig. 126 for $\angle B$.

$$\frac{45}{\sin B}=\frac{38}{\sin 54°\ 30'}$$

$$\sin B=\frac{45\times\sin 54°\ 30'}{38}$$

$$\sin B=0.84464$$

$$\sin 57°\ 38'=0.84464$$

$$\angle B=57°\ 38'$$

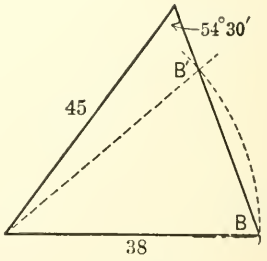


Fig. 126.

NOTE. Two triangles are possible the one as shown in heavy lines, and the one as shown in dotted lines. The value of $\angle B$ in the second triangle is

$$180° - 57°\ 38';$$

since

$$\sin B = \sin (180 - B).$$

EXERCISE 1

Find the unknown parts in each of the following triangles:

1. Fig. 127. Ans. $\angle C = 112°\ 31'$.
$x = 12.913$.
$y = 31.593$.

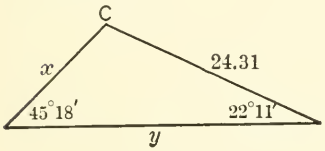


FIG. 127.

2. Fig. 128. Ans. $\angle B = 46°\ 36'$.
$M = 2362.6$.
$N = 2051.5$.

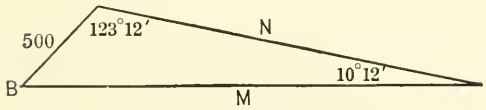


FIG. 128.

3. Fig. 129. Ans. $\angle B = 37°\ 58'$.
$\angle A = 65°\ 2'$.
$d = 929.48$.

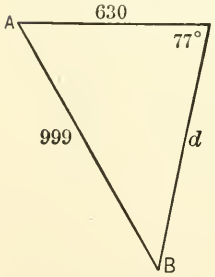


FIG. 129.

139. Law of Cosines. In Fig. 124,

$$b^2 = \overline{AD}^2 + h^2$$

$$\overline{AD} = c - \overline{DB}$$

$$\overline{AD}^2 = c^2 - 2c\overline{DB} + \overline{DB}^2$$

$$b^2 = c^2 - 2c\overline{DB} + \overline{BD}^2 + h^2$$

$$DB^2 + h^2 = a^2$$

$$\frac{\overline{DB}}{a} = \cos B; \quad \overline{DB} = a \cos B$$

Therefore:

$$b^2 = c^2 + a^2 - 2ac \cos B$$

Similarly,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

and

$$c^2 = a^2 + b^2 - 2ab \cos C$$

These three formulas have the same form and meaning and are known as the law of cosines. One form must be memorized.

Example 1. Solve the triangle, Fig. 130, for x.

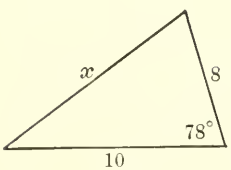


Fig. 130.

$$x^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 78°$$

$$x^2 = 64 + 100 - 2 \times 8 \times 10 \times .20791$$

$$x^2 = 64 + 100 - 33.264$$

$$x^2 = 130.736$$

$$x = 11.433$$

Example 2. Solve the triangle of Fig. 131 for angle A.

$$6^2 = 5^2 - 2\times 5\times 7 \cos A$$

$$36 = 25 + 49 - 70 \cos A$$

$$70 \cos A = 38$$

$$\cos A = .5429$$

$$\angle A = 57^\circ\ 7'$$

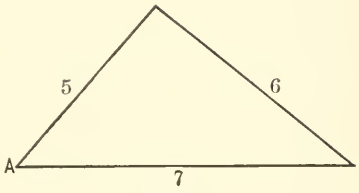


Fig. 131.

EXERCISE 2

Solve the following triangles:

1. Fig. 132.

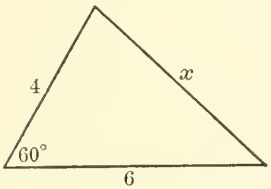


Fig. 132.

Ans. $x = 5.2915$.

2. Fig. 133.

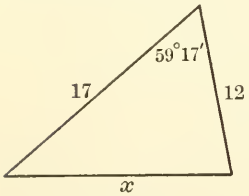


Fig. 133.

Ans. $x = 14.987$.

3. Fig. 134.

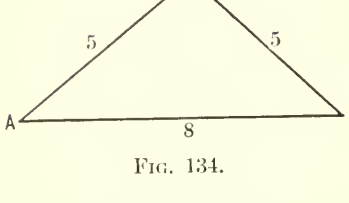


Fig. 134.

Ans. $\angle A = 36° \ 52'$.

140. Law of Tangents. In some cases it is more efficient to use a relation between the sides and angles of a triangle known as the **law of tangents.**

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$. See Fig. 135.

The proof is omitted.

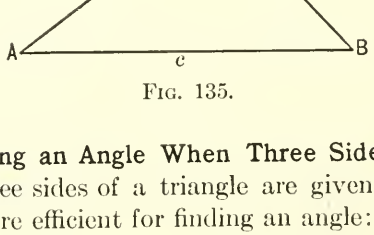


Fig. 135.

141. Finding an Angle When Three Sides Are Given. When the three sides of a triangle are given, the following formula is more efficient for finding an angle:

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

where a, b, and c are the sides of the triangle and $s = \frac{1}{2}(a+b+c)$.

142. Areas of Triangles. The area of a triangle is expressed by the formula $S=\frac{1}{2}bh$, where b is the base and h is the altitude.

When two sides and the included angle are given (Fig. 136):

$$\frac{h}{a}=\sin C$$

$$h=a\sin C$$

Therefore

$$S=\tfrac{1}{2}ab\sin C$$

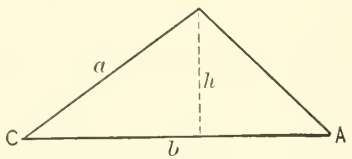


FIG. 136.

When two angles and the included side are given (Fig. 137):

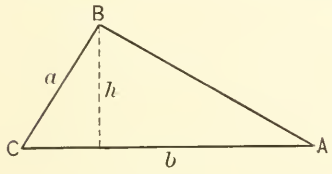


FIG. 137.

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

$$a=\frac{b\sin A}{\sin B}$$

Substituting in the formula of previous method:

$$S=\frac{1}{2}\frac{b^2\sin A\sin C}{\sin B}$$

$$B=180-(A+C)$$

$$\sin B=\sin(180-B)=\sin(A+C)$$

Therefore,

$$S=\frac{1}{2}\frac{b^2\sin A\sin C}{\sin(A+C)}$$

Problems falling under the two above cases can be solved also by use of the law of sines and right triangulation.

When three sides are given:

The following formula is given where a, b, and c are the three sides and $s=\frac{1}{2}(a+b+c)$,

$$S=\sqrt{s(s-a)(s-b)(s-c)}$$

The proof can be worked from the law of cosines and $\sin^2 x+\cos^2 x=1$.

EXERCISE 3

Find the area of the following triangles:

1. Fig. 138.

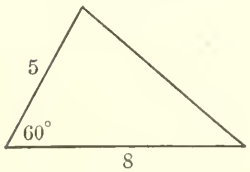


Fig. 138.

Ans. 17.320.

2. Fig. 139.

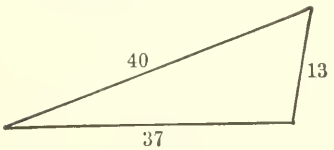


Fig. 139.

Ans. 240.

3. Fig. 140.

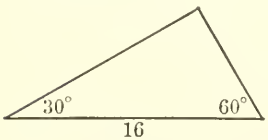


Fig. 140.

Ans. 55.427.

4. Fig. 141.

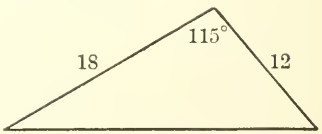


Fig. 141.

Ans. 97.880.

CHAPTER XVI

ELECTRICAL APPLICATIONS

143. Projection. The *projection* of one line upon another line is the portion of that second line included between the

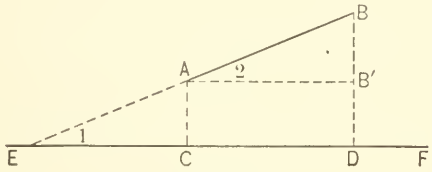


Fig. 142.

perpendiculars dropped upon it from the extremities of the first line. The projection of AB upon EF Fig. 142, is CD.

The projection of one line upon another is found by multi-

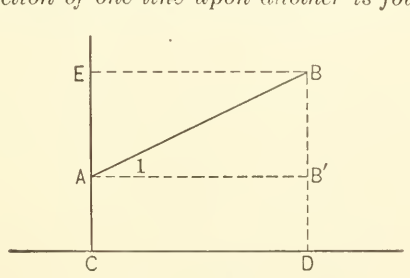


Fig. 143.

plying the first line by the cosine of the angle between the two lines. (Angle 1 is the angle between AB and EF.)

Proof: $\cos \angle 2 = \dfrac{AB'}{AB}$

$AB \cos \angle 2 = AB'$

$\angle 1 = \angle 2$ (Corresponding angles)

$AB' = CD$ (Parallels between parallels)

$AB \cos \angle 1 = CD$ (Substitution)

EXERCISE 1

(Refer to Fig. 143)

1. If $AB = 8''$, $\angle 1 = 35°$, find CD. Ans. 6.5536.
2. If $AB = 14''$, $\angle 1 = 75°$, find CD. Ans. 3.6232.
3. If $\angle 1 = 30°$, how long must AB be to make $CD = 10''$. Ans. 11.545.
4. If $AB = 12''$, and $CD = 9''$, find $\angle 1$. Ans. 41° 24′.
5. If $\angle 1 = 40°$, and $AB = 12''$, find AE. Ans. 7.7136.

NOTE. $AE = BB'$.

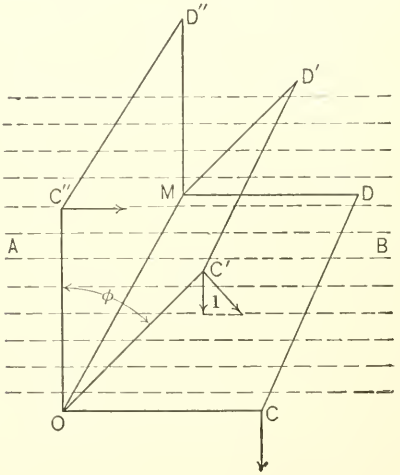


FIG. 144.

144. Lines of Force Cut by a Loop. AB, Fig. 144, represents a field of magnetic flux or lines of magnetic force. $OCDM$ is a loop of wire rotating about the line OM. The wires OC, MD, and OM do not cut any lines of force in any position of rotation. At the position OC'', CD is not cutting any lines, but at the position OC', CD is cutting some lines, and at the position OC, CD is cutting lines at its maximum rate.

EXERCISE 2

1. Suppose CD in the position shown by OC, Fig. 144, is cutting lines at the rate of 120. At what rate will it be cutting lines when $\angle \phi = 50°$, if the angular velocity remains the same? Ans. 91.92.

2. Taking OC'' as the starting position and E as the maximum rate of cutting lines (position OC) find the formula for the rate of cutting lines at any position OC', in terms of ϕ and E. Call this rate e. Ans. $e = E \sin \phi$.

145. Voltage Generated by a Rotating Loop. A voltage will be generated in the loop $OCDM$, Fig. 144 proportional to the rate at which CD cuts the lines of force. The maximum voltage is obtained in the position OC ($\phi = 90°$) and the minimum voltage in the position OC''($\phi = 0°$). The voltage at any position OC' is called the **instantaneous voltage,** and is expressed by the formula derived in problem 2 above.

EXERCISE 3

1. If the maximum voltage is 10, compute the instantaneous voltage for every 10° up to 360° Tabulate the results.

2. Plot a curve of the results obtained in Problem 1 with the values of ϕ as abscissas and the instantaneous

voltages as ordinates. (This is a sine or voltage curve. Save it for future use.)

3. From the curve of Problem 4 read the value of the voltage at 45°, 135°, 225° and 315°.

Ans. 7.1, 7.1, −7.1, −7.1.

146. Phase Angle. The value of the instantaneous voltage can be shown also by revolving a line, of a length representing the maximum voltage, about a point. The angle between the line and the X-axis is the angular position ϕ, and is called the **phase angle.** The projection upon the Y-axis is the instantaneous voltage ($=E \sin \phi$). The diagram is arranged as Fig. 145.

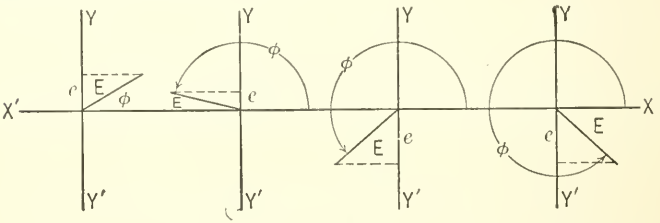


Fig. 145.

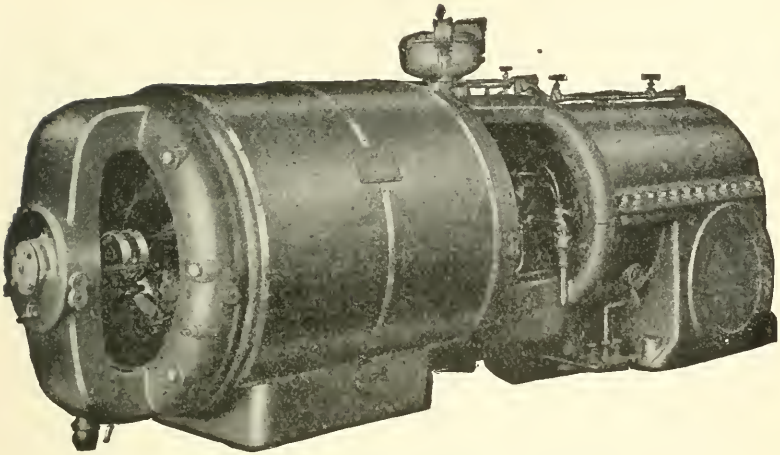
EXERCISE 4

1 With scale and protractor compute the instantaneous voltage if $E = 10$, when $\phi = 25°$, 80°, 120° and 325°.

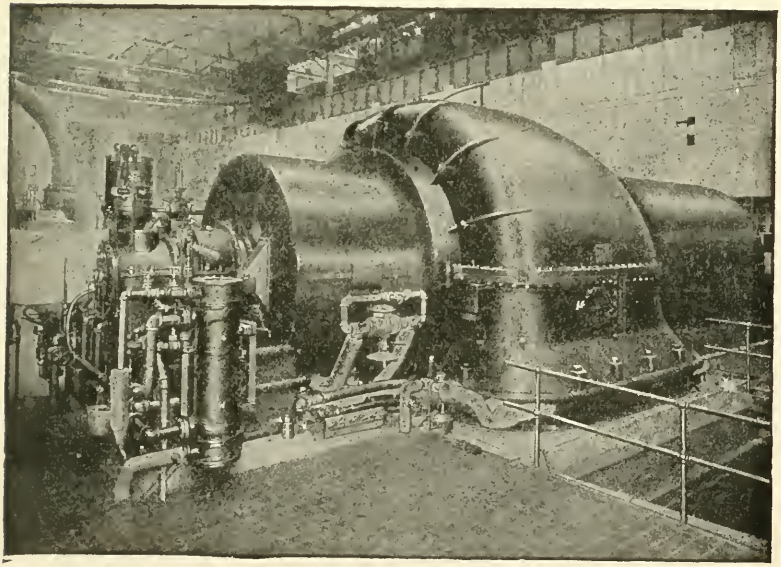
Ans. 4.2, 9.8, 8.7, −5.7.

2. On the same axis and scale as used in Problem 2, Exercise 3, plot another curve where $E = 8$ and ϕ is 0 when the ϕ of the other curve is 30°. (This curve will represent a voltage curve of a loop similar to *OCDM* but 30° behind it and not as large a loop.)

3. If the two loops are connected in series and rotated, the combined voltage at any time will be the sum of the two



300-Kilowatt Three-Phase Turbo-Alternator.



35,000-Kilowatt Three-Phase Turbo-Alternator.

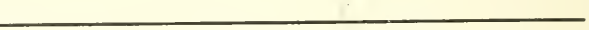
instantaneous voltages. Plot a curve of such sums on the same scale and axis as used in Problem 2. (Use the compass or dividers to lay off the sums, and save reading the numerical values.) (This curve is the resultant curve of the sum of the other two curves.)

Compare the maximum of the resultant curve with the sum of the maximum of the other two curves. Can you see from the two curves why these two quantities are not equal?

147. Vectors. The method of Problem 3 above is one of the methods of adding two A.C. voltages. The two voltages can also be added by vector addition.

Some physical quantities can be represented completely by means of a straight line. The length of the line can be used to represent the numerical value of the quantity and the direction of the line to represent the sense of the quantity. For example, a distance of 3 miles can be represented by a line 3″ long, and, if it is a distance to the east, that fact can be represented by making the line point to to the right by means of an arrow head thus:

Then a line 5″ long and pointing in the opposite direction thus



will represent a distance of 5 miles to the west. Such quantities are called **vector quantities.** A straight line representing a vector quantity is called a **vector.**

148. Scalar Quantities. Quantities which can be expressed completely by stating their magnitude are called **scalar quantities.** An example of a scalar quantity is 5 bushels or 7 ohms, etc.

149. The Use of Vectors. Many quantities in electricity can be expressed by vectors much more clearly than by words. Vectors form a valuable method for adding and subtracting certain quantities, and are used in all texts on electricity. Therefore it is necessary to study methods of expressing quantities by vectors and interpreting the meaning of quantities expressed by vectors.

Example. A man travels 4 miles east and then 3 miles north. How far and in what direction is he from the starting point. Solve by vectors.

Solution.

The line AC, Fig. 146 represents 4 miles to the east and the line CB represents 3 miles to the north. The line AB represents the single equivalent journey. AB is called the **vector sum** of AC and CB. If AB is measured to the same scale as AC and CB, it will be found to represent 5 miles, and if the angle A is measured with a protractor it will be found to be about 37°. Therefore the vector AB is interpreted as representing a distance of 5 miles in the direction of 37° to the north of east.

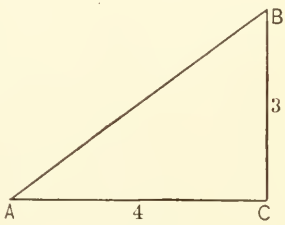


FIG. 146.

EXERCISE 5

Solve by vectors:

1. A man travels 10 miles south then 12 miles east. How far and in what direction is he from the starting point?

2. A man travels 10 miles west then 2 miles north and 8 miles southeast. How far and in what direction is he from the starting point.

150. The Composition of Forces. Another and more important use made of vectors is to study the effect of two forces acting upon the same object.

Example. An object is acted upon by two forces, one of 6 lbs. and one of 8 lbs. acting at an angle of 90°, Fig. 147. What is the value and direction of the single force that would produce the same effect?

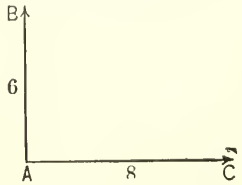


FIG. 147.

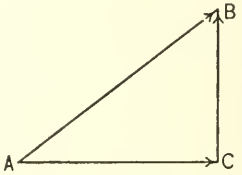


FIG. 148.

By a law of mechanics a force produces the same effect upon an object whether it acts independently or in connection with other forces. Therefore, AB and CD acting together would produce the same effect as if AC acted and then AB acted. This could be represented as in Fig. 148. Then AB represents the value and direction of the single resultant force. Or taking the forces as shown in Fig. 147 and completing a parallelogram as Fig. 149 the diagonal AD also represents the resultant. AB of Fig. 148 $= AD$ of Fig. 149, since $CD = AB$ (opposite sides of a parallelogram).

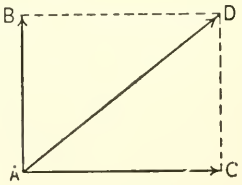


FIG. 149.

EXERCISE 6

1. Add a force of 5 lbs. to a force of 12 lbs. acting at right angles to it.

2. Add a force of 8 lbs. to a force of 12 lbs. acting at an angle of 60°.

Hint. Draw the vectors at an angle of 60° and complete the parallelogram, etc.

3. Add a force of 3 lbs. acting horizontally and a force of 4 lbs acting at an angle of 20° above horizontal, and a force of 2 lbs. acting at an angle of 50° above the horizontal.

Hint. Three or more vectors can be added by adding two and then the third one to that sum, etc., or they may be added by being placed end on end in the proper direction as Fig. 150.

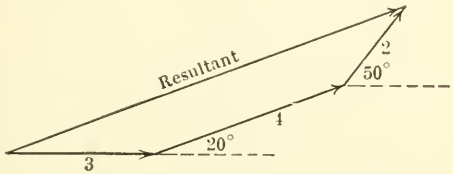


Fig. 150.

4. Add a force of 5 lbs. acting horizontally, and a force of 6 lbs. acting at an angle of 37° above the horizontal and a force of 4.5 lbs. acting at an angle of 60° above the horizontal.

151. Subtraction of Vectors. From algebra it is known that subtracting a number is the same as adding an equal number with the opposite sign, thus:

$$4-(+2)=4+(-2).$$

Similarly, in the subtraction of vectors, to subtract vector 1 from vector 2, add to vector 2 a vector (3) equal to vector 1 but of opposite direction, as Fig. 151.

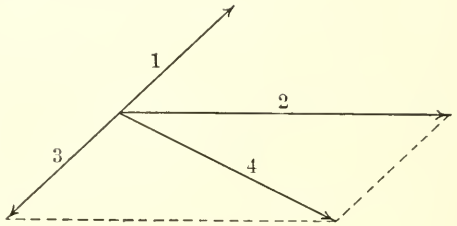


Fig. 151.

Vector 4 then is the difference between vectors 1 and 2.

Subtraction of vectors can be arranged more conveniently as in Fig. 150. Complete the parallelogram for adding vectors 1 and 2.

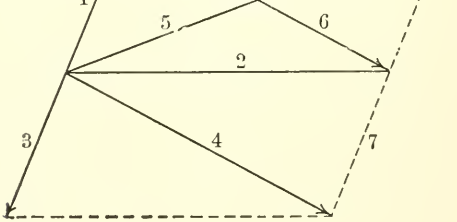


Fig. 152.

Vector 5 represents the sum of 1 and 2. Vector 6 is equal and parallel to vector 4, and therefore represents the difference between 1 and 2.

Proof. Vector 3 = Vector 1 (Construction)
Vector 3 = line 7 (Opposite sides of a parallelogram)
Therefore, Vector 1 = line 7
Vector 1 is parallel to line 7 (Const.)

Therefore lines 1, 4, 7, 6 form a parallelogram.

Therefore vector 6 is equal to, and parallel to vector 4, hence vector 6 represents the vector difference between 1 and 2.

Rule. *When two vectors are drawn from a point and the parallelogram is completed, the diagonal of the parallelogram, drawn from the point is the vector sum of the two vectors and the other diagonal is the vector difference.*

NOTE. The vector difference may be directed in either direction, depending upon the order in which the vectors are subtracted.

EXERCISE 7

1. Add by vectors a force of 40 lbs. and a force of 60 lbs. acting at an angle of 30°.

NOTE. Alternating current voltages may differ in phase, that is in the time of reaching maximum or minimum value. Phase is expressed as an angle which corresponds to the difference in position of two coils on the armature. Alternating current voltages can be represented by vectors, the phase angle is generally measured from a horizontal line to the vector.

2. Add by vectors a voltage of 8 volts, phase angle 15°, to a voltage of 10 volts, phase angle 45°. Compare the sum with the maximum resultant from the curves of Problem 3, Exercise 4.

3. Check the result of Problem 2 by solving the triangle by trigonometry.

4. Add by vectors 12 volts, phase angle 10°, 15 volts, phase angle 35° and 8 volts, phase angle 20°. Measure magnitude, phase angle, and instantaneous value of the resultant. (See page 180.)

5. Solve No. 4 by trigonometry.

6. Study the vector diagram of Problem 4 to see what can be read from it in regard to instantaneous values of the separate voltages, etc.

7. Circuits of 12 volts, phase angle 20° and 20 volts, phase angle 45° are connected in series. Find the vector sum and difference.

8. A circuit has 110 volts, phase angle 36°. It is desired to make the circuit contain 150 volts, phase angle 2.8°. How many volts must be added and in what phase.

9. Solve No. 8 by trigonometry.

152. Current. Current is proportional to voltage, therefore, the current curve is similar to the voltage curve and is expressed by the formula

$$i = I \sin \phi,$$

where i is the instantaneous current and I the maximum current.

If the circuit contains no inductance or capacity, the current will be in phase with the voltage. That is, the current will be zero when the voltage is zero, and maximum when the voltage is maximum.

If the circuit contains inductance only, the current will lag 90° behind the vo tage.

If the circuit contains capacity only, the current will lead the voltage by 90°. Most alternating current circuits contain resistance, inductance and capacity. The current is not ordinarily in phase with the voltage.

EXERCISE 8

1. Draw roughly a current curve in phase with its voltage curve.

2. Draw roughly a current curve leading its voltage curve by 90°.

3. Draw roughly a current curve lagging behind its voltage curve by 90°.

4. Draw roughly a current curve lagging behind its voltage curve by 30°.

The above exercises can be represented vectorially as in Fig. 153.

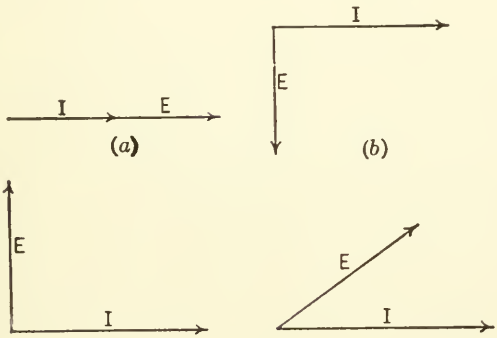
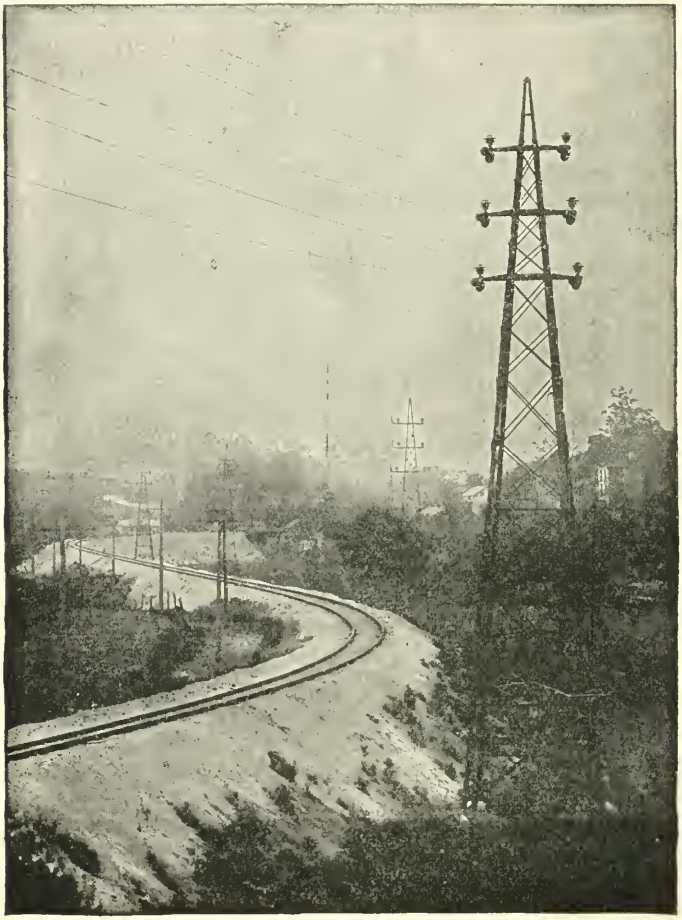


FIG. 153.

Fig. 153 (*a*) represents current in phase with the voltage. Fig. 153 (*b*) represents the current leading the voltage by 90°. Fig. 153 (*c*) represents the current lagging 90° behind the voltage. Fig. 153 (*d*) represents the current lagging 30° behind the voltage.

153. Power. From the curves of Exercise 8, it is evident that, at any instant, the power is equal to the *instantaneous* current times the *instantaneous* voltage.



Double Circuit Three-Phase Transmission Line with Pin Type Insulators.

The current and voltage do not reach a maximum at the same time, hence maximum power is not maximum current times maximum voltage. Similarly, effective power is not equal to effective current times effective voltage, but, *the power at any instant is equal to the current times the component of the voltage that is in the same phase with the current.* Graphically, this can be represented as in Fig. 154.

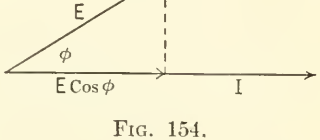


Fig. 154.

Power, then, is represented by the formula

$$P = EI \cos \phi,$$

where P is the maximum of effective power, E is the maximum or effective voltage and I is maximum or effective current. ϕ is the phase angle between the current and voltage, and cos ϕ is called the **power factor.**

EXERCISE 9

1. If $I = 10.5$, $E = 125$, and $\phi = 40°$ find the power.
2. In Problem 1 what is the power factor?

Ans. .7660.

3. In an A.C. circuit the current lags 40°, the voltmeter indicates 120 volts, the ammeter 16 amperes. What would a wattmeter read?
4. An ammeter in a 120 volt A.C. circuit indicates 8 amperes. A wattmeter in the same circuit reads 500 watts. Find the power factor and phase difference.
5. A 110 volt A.C. generator is to deliver 2100 watts to a circuit having a power factor of 0.84. What current is necessary?

154. Force on a Conductor Carrying Current in a Magnetic Field. The force exerted on a conductor carrying current and lying in a magnetic field is expressed by the equation

$$F = \frac{IlH \sin \phi}{10},$$

where F = force in dynes on the conductor;
I = current through the conductor;
l = length of the conductor in centimeters;
H = strength of the magnetic field in gausses,
ϕ = angle between the conductor and the magnetic field.

EXERCISE 10

1. Find the force on a conductor 45 cm. long carrying a current of 12 amperes in a field of 16,000 gausses, if the angle between the conductor and the lines of magnetic force is 80°. Ans. 850,860.

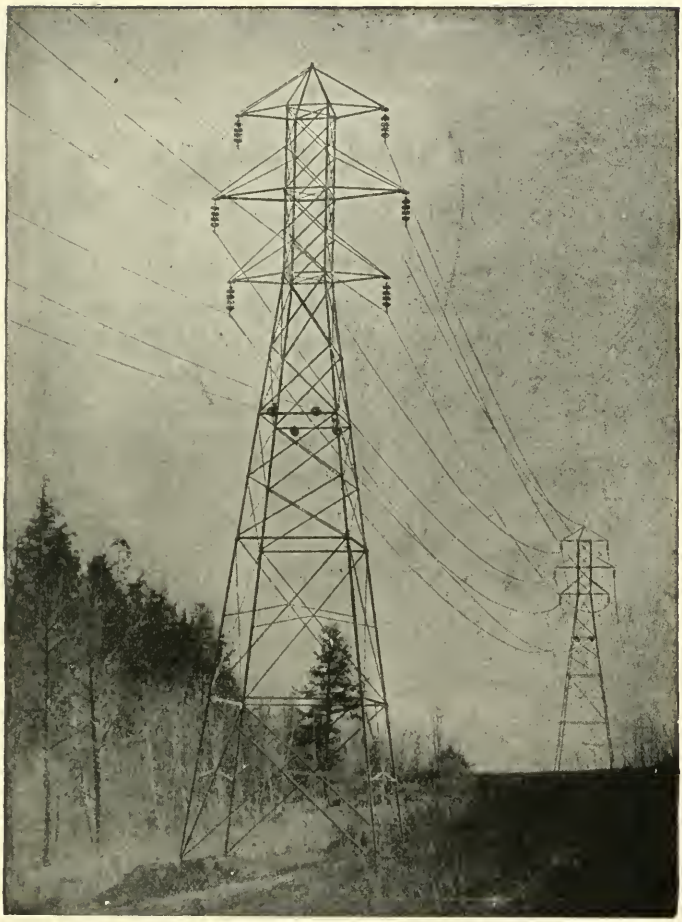
2. How great a magnetic flux will be required to exert a maximum force ($\phi = 90°$) of 1,250,000 dynes on a conductor 18″ long carrying a current of 5 amperes? Ans. 54,680.

3. Find ϕ when $F = 2{,}350{,}000$, $I = 8$, $l = 2'$, $H = 125{,}000$. Ans. 22° 40′.

155. Impedance. Impedance in an alternating current circuit may be computed from the formula

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2},$$

where Z = impedance;
R = resistance of circuit in ohms;
f = number of cycles per second;
L = inductance in henries;
C = capacity in farads.



Double Circuit Three-Phase Transmission Line with Suspension Insulators.

EXERCISE 11

1. Compute Z when $R=230$, $f=60$, $L=.15$, $C=.0003$. Ans. 235.1.

2. When $L=.122$ and $C=.00024$, what value of f will make $\left(2\pi fL-\frac{1}{2\pi fC}\right)=0$? Ans. 29.41.

3. When $C=.0004$ farad and $f=500$, what value of L will make $\left(2\pi fL-\frac{1}{2\pi fC}\right)=0$? Ans. .00025.

NOTE. High frequency currents are used in radio work.

156. Computation of Current. Current in an alternating current circuit may be found from the equation

$$I=\frac{E}{\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}},$$

where $I=$ current;
$E=$ voltage.

EXERCISE 12

1. Find the current that will flow through a circuit of 125 ohms, .02 henry, and .00003 farad, when a 60-cycle 120-volt alternating-current voltage is applied. Ans. .806.

MISCELLANEOUS PROBLEMS

The problems which follow are selected to give to the student, as nearly as possible, practical experience in computation. The problems are not classified but cover a wide and general field. Logarithms should be used in the computations.

1. A distance AB of 154.59′, Fig. 155, is measured along the bank of a river. A tree C is sighted on the opposite bank, making the angle at A equal to 34° 28′, and the angle at B equal to 90°. Find the width of the river BC.

Ans. 106.11 ft.

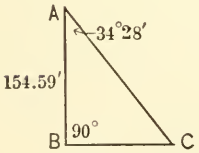


Fig. 155.

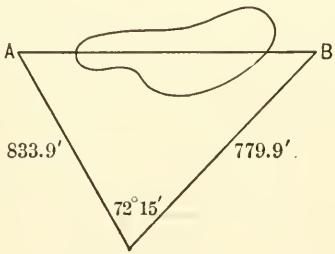


Fig. 156.

2. To find the distance between A and B separated by a lake, Fig. 156, readings were made as shown.

Find the distance AB. Ans. 952.4.

3. Two sides of a triangular lot are found to be 173 rods and 194 rods. The included angle is 56° 15′. Find the area. Ans. 87.2 acres.

4. Eight holes are to be drilled in a circle of radius 5 in. How far apart are the holes? Ans. 3.8267.

5. To drill the holes in the above problem it will be necessary to move the table, as in Fig. 157.

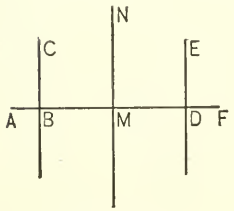
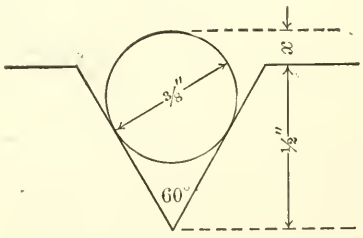


Fig. 157.

Find	Ans.
AB.	1.464.
BC.	3.536.
AM.	5.
MN.	5.
AD.	8.537.
DE.	3.536.
AF.	10.

6. The latitude of Washington, D. C., is 38° 55′ N. How many miles east and west on the earth's surface make a difference in time of 1 hour in the latitude of Washington? Radius of the earth is 3957 miles. Ans. 806.02.

7. Find x, Fig. 158.



Ans. 0.0625.

Fig. 158.

8. Find x, Fig. 159 (angles are 90°). Ans. 4.0588.

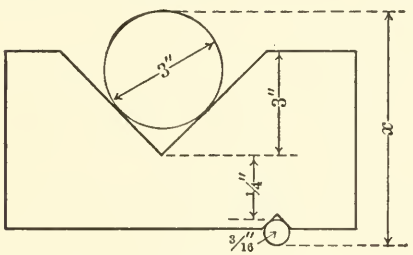


Fig. 159.

9. The angle of elevation of the top of a mountain, observed from a point directly south of it is 60°. From a point 1 mile directly east of the first point and on the same level with it the angle of elevation is 45°. Find the height of the mountain. Ans. 6466.8.

10. A light-house was observed to bear directly east from a ship. After the ship sailed 4 miles north the light-house bore 55° 30′ east of south. How far was the ship from the light-house at the time of each observation? Ans. 5.8202, 7.062.

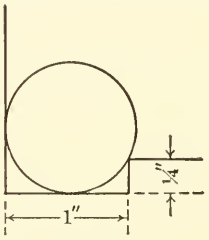


Fig. 160.

11. One side of a hexagon is 2 in. Find the area. Ans. 10.3926.

12. Find the radius of the circle, Fig. 160. Ans. .5429.

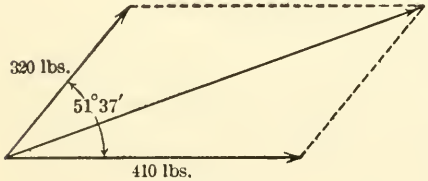


Fig 161.

13. If two forces of 410 lbs. and 320 lbs. pull at an angle of 51° 37′, as shown in Fig. 161, the line AB, which is the diagonal of the paral-

lelogram of which AM and AN are the sides, represents the value and direction of the resultant force. Find the value of the resultant force and the angle which it makes with the 410-lb. force. Ans. 658.36 lbs., 22° 23′ 47″.

14. An unknown force combined with a force of 128 lbs. produces a resultant of 200 lbs. and this resultant makes an angle of 18° 24′ with the 128-lb. force. Find the intensity and direction of the unknown force.

Ans. 88.326 lbs., 54° 37′ 16″.

15.

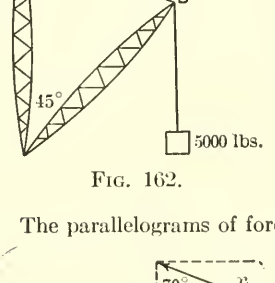


Fig. 162.

Find the tension in the cable AB, Fig. 162.

Ans. 3901 lbs.

Note. The parallelograms of forces, Fig. 163, can be used.

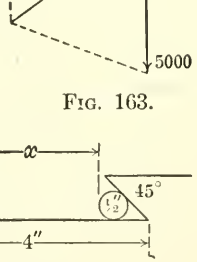


Fig. 163.

16.

Fig. 164.

Find the distance x between the plugs, Fig. 164.

Ans. 2.2930 in.

17.

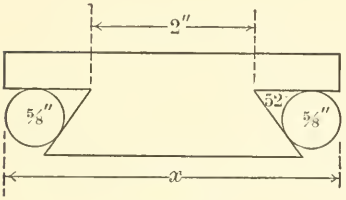


Fig. 165.

Find the distance x, Fig. 165.

Ans. 3.9065.

18. From the top of a hill the angles of depression of two objects 5280 ft. apart on a straight level road leading to the hill are 5° and 15°. Find the height of the hill.

Ans. 685.9 ft.

APPENDIX

DICTIONARY OF THE TERMS USED IN ELEMENTARY MATHEMATICS

Abscissa. The horizontal or X coordinate of a point.

Acute Angle. An angle of less than 90°.

Adjacent. Next to or adjoining.

Adjacent Angles. Angles having a common side and a common vertex.

Altitude. Height measured perpendicular to the base.

Angle. The difference in direction of two lines that meet at a point.

Angle of Depression. The angle made by the line of sight to an object below the horizontal plane of the observer, and the plane.

Angle of Elevation. The angle made by the line of sight to an object above the horizontal plane of the observer, and the plane.

Antecedent. The first or third term of a proportion.

Antilogarithm. The number corresponding to a given logarithm.

Apothem. The perpendicular distance from the center of a regular polygon to one of the sides.

Arc. Part of the circumference of a circle.

Axiom. A truth that is self-evident.

Base. The side or surface upon which a geometrical figure appears to rest.

Base. An expression which is to be raised to a power.

Binomial. An expression having two terms.

Bisector. A line or plane which divides a geometrical figure into two equal parts.

Central Angle. An angle which has its vertex at the center of a circle.

Characteristic. The whole number part of a logarithm.

Check. Proof of the solution of a problem.

Chord. A straight line connecting two points on the circumference of a circle.

Circle. A plane figure bounded by a curved line every point of which is equidistant from a point called the center.

Circular Mil. A unit of area equal to the area of a circle one mil in diameter.

Circumference. The curved line forming a circle.

Circumscribed. Drawn about, as a polygon circumscribed about a circle.

Coefficient. The coefficient of a factor is the product of all the remaining factors.

Cologarithm. The logarithm of the reciprocal of a number.

Commensurable Quantities. Quantities whose common unit of measure is a rational quantity.

Complement. The angle which will add to a given angle to make 90°.

Concentric. Having the same centers.

Cone. A solid generated by the rotation of a right triangle about one of its legs.

Congruent. Equal in all respects.

Consequent. The second or fourth term of a proportion.

Constant. A quantity whose value does not change.

Continued Proportion. A proportion of three or more ratios.

Converse. Reversed in the order of relation.

Coordinate. The distance of a point from the coordinate axes.

Coordinate Axes. The reference lines by which a graph is plotted.

Corollary. A truth which follows naturally from another truth.

Corresponding Parts. Parts similarly placed in congruent or similar geometric figures.

Cube. A rectangular solid having all of its faces squares.

Cube. The third power of a number.

Cutting Speed. The speed at which work passes the point of a cutting tool.

Cylinder. The solid generated by revolving a rectangle about one of its sides.

Definite Number. A number having always the same value.

Degree. $\frac{1}{360}$ of the angular space about a point.

Diagonal. A line connecting any two non-adjacent vertices of polygon or polyhedron.

Diameter. A line passing through the center of a circle and ending at the circumference.

Dihedral Angle. The angle formed by two intersecting planes.

Ellipse. Section of a cone made by a plane cutting the cone not parallel to or cutting the base.

Equation. A statement that two expressions are equal.

Equiangular. A plane figure having equal angles.

Equilateral. A plane figure having equal sides.

Equivalent. Equal in area or volume.

Evaluation. The process of substituting definite numbers for general numbers and performing the operations indicated.

Exponent. A small number written above and to the right of a number (called the base) to show how many times the base is to be used as a factor.

Exterior Angle. The angle between one side of a polygon and an adjacent side extended.

Extremes. The first and fourth terms of a proportion.

Factor. One of the quantities which, multiplied together, form a given product.

Formula. Statement of a rule or principle in terms of general numbers.

Frustum. The part of a cone or a pyramid next the base, formed by cutting off the top.

Fulcrum. The point about which a lever turns.

Function. A quantity that depends upon another quantity for its value.

Function of an Angle. Sin, cos, or tan, etc., of the angle.

General Number. A number that has different values in different problems.

Graph. A curve representing the relation between two variables.

Great Circle. The largest circle that can be drawn on the surface of a sphere.

Hexagon. A plane figure having six sides.

Hypotenuse. The side of a right triangle opposite the right angle.

Incommensurable Quantities. Quantities whose common unit of measure is an irrational quantity.

Index. The number above and to the left of the radical sign which indicates what root is to be taken.

Index. The end of a slide rule scale.

Inscribed. Drawn within; a polygon is inscribed in a circle when all the vertices of the polygon lie on the circumference of the circle.

Inscribed Angle. An angle whose vertex lies on circumference of a circle and whose sides are chords of the circle.

Intercept. To cut.

Intercept. The part included between two points.

Interpolation. The process of finding intermediate terms from a table by means of the two nearest terms given.

Irrational Quantity. A quantity whose value cannot be expressed exactly by decimals or fractions, as $\sqrt{2}$.

Isosceles Triangle. A triangle having two sides equal.

Least Common Denominator. The smallest number that will contain two or more denominators evenly.

Lever. A rigid piece capable of turning about a point called a fulcrum.

Leverage. Tendency to turn, turning moment.

Lever Arm. Distance from the fulcrum of a lever to a force acting upon the lever, measured perpendicular to the direction of the force.

Locus. The path of a point or curve moving according to some law.

Logarithm. The exponent of that power of a base (usually 10) which equals a given number.

Lune. A portion of the surface of a sphere bounded by the semicircumferences of two great circles.

Mantissa. The decimal part of a logarithm.

Mean Proportion. When the second and third terms of a proportion are the same the proportion is a mean proportion.

Mean Proportional. The second (or third) term of a mean proportion.

Means. The second and third terms of a proportion.

Median. A straight line drawn from the vertex of a triangle to the midpoint of the opposite side.

Member. One of the parts of an equation separated by the equal sign.

Mil. One thousandth of an inch.

Monomial. An expression having only one term.

Negative Number. A number to be subtracted, or a number on the number scale to the left of the zero.

Oblique Angle. Any plane angle not a right angle, straight angle, or perigon.

Oblique Triangle. A triangle having all oblique angles.

Obtuse Angle. An angle greater than a right angle and less than a straight angle.

Octagon. A plane figure having eight sides.

Ordinate. The vertical or Y coordinate of a point.

Parallel Lines. Straight lines in the same plane that will not meet if extended.

Parallelogram. A quadrilateral having its opposite sides parallel.

Parallelopiped. A six-sided solid all of whose faces are parallelograms.

Pentagon. A plane figure having five sides.

Perigon. An angle of 360°.

Perimeter. The distance around a plane figure.

Perpendicular. At right angles to a line or surface.

Phase. Position in a cycle of rotation or oscillation.

Place. The position of a figure in a number.

Plane. A surface without curvature.

Polygon. A plane figure bounded by straight lines.

Polyhedral Angle. The solid angle formed by three or more planes meeting at a point.

Polyhedron. A solid bounded by planes.

Polynomial. An expression having two or more terms.

Positive Number. Ordinary or arithmetical number.

Power. Product of equal factors.

Prism. A solid whose bases are congruent and parallel and whose sides are parallelograms.

Projection. (Of a point on a line.) The foot of the perpendicular from that point to the line. (Of a line or surface on a line or surface.) The line or surface join-

ing the projection of all points of that line or surface, on the second line or surface.

Proportion. When two ratios are equal, they form a proportion.

Protractor. An instrument for drawing and measuring angles.

Pyramid. A solid having for its base a polygon and for its sides triangles meeting at a common point.

Quadrant. One-fourth of a circle or perigon.

Quadrilateral. A plane figure having four sides.

Radius. A straight line from the center of a circle to the circumference, or from the center of a sphere to the surface.

Ratio. The relation of one quantity to another quantity of the same kind.

Rational Quantity. A quantity whose value can be expressed exactly by decimals or fractions.

Reciprocal. One divided by a given number.

Rectangle. A parallelogram having right angles.

Rectilinear. Having straight lines.

Reflex Angle. An angle greater than 180°.

Regular Polygon. A polygon that has its sides equal and its angles equal.

Resultant. The single force replacing two or more forces. Also applies to other vectors.

Rhomboid. A parallelogram having its angles oblique and its adjacent sides unequal.

Rhombus. A parallelogram having its angles oblique and its sides equal.

Right Angle. An angle of 90°.

Right Prism. A prism whose sides are perpendicular to the base.

Right Triangle. A triangle having one right angle.

Rim Speed. The speed of a point on the surface of a revolving object.

Runner. The sliding glass piece on a slide rule.

Scalar Quantity. A quantity which can be expressed completely by giving its magnitude. Sometimes called a scalar.

Scalene Triangle. A triangle having no two sides equal.

Secant. A straight line passing through a circle.

Sector. The portion of a circle between two radii.

Segment. The portion of a circle between a chord and the circumference.

Similar. Of the same shape.

Slide. The central portion of a slide rule.

Slide Rule. A calculating instrument.

Solid. Any object having definite shape.

Sphere. A solid bounded by a curved surface every point of which is equidistant from the center.

Square. The second power of a number.

Square. A rectangle having equal sides.

Square Root. The number which when multiplied by itself equals a given number.

Straight Angle. An angle of 180°.

Substitution. The process of replacing one quantity by another.

Subtend. Cut off.

Supplement. The angle which added to a given angle will make 180°.

Symmetrical. The same on both sides of a point line, or plane.

Tangent. Touching but not cutting.

Tetrahedron. A polyhedron having four equal faces.

Term. An expression whose parts are not separated by the plus or minus sign.

Theorem. A statement to be proved.

Torque. Forces tending to produce rotation.

Transpose. To change a term from one member of an equation to the other.

Transversal. A line cutting other lines.

Trapezium. A quadrilateral having no two sides parallel.

Trapezoid. A quadrilateral having one pair of parallel sides.

Triangle. A plane figure having three sides.

Variable. A quantity that changes in value.

Vector. A line representing a vector quantity.

Vector Quantity. A quantity having magnitude and direction or sense.

Vertical Angles. If two lines intersect, the opposite angles are vertical angles.

RELATIONS BETWEEN THE TRIGONOMETRIC FUNCTIONS

Certain relations exist between the functions of angles. These relations are used in finding one function from other functions. The following table is given for reference. The proofs of the relations are given in books more complete in the theory of trigonometry:

1. $\sin^2 x+\cos^2 x=1.$
2. $\dfrac{\sin x}{\cos x}=\tan x.$
3. $\dfrac{\cos x}{\sin x}=\cot x.$
4. $\sec^2 x=1+\tan^2 x.$
5. $\csc^2 x=1+\cot^2 x.$
6. $\sin x=\dfrac{1}{\csc x}.$
7. $\cos x=\dfrac{1}{\sec x}.$
8. $\tan x=\dfrac{1}{\cot x}.$
9. $\sin\ (x+y)=\sin x\ \cos y+\cos x\ \sin y.$
10. $\cos\ (x+y)=\cos x\ \cos y-\sin x\ \sin y.$
11. $\tan\ (x+y)=\dfrac{\tan x+\tan y}{1-\tan x\ \tan y}.$

12. $\cot\,(x+y)=\dfrac{\cot x\cot y-1}{\cot x+\cot y}$.

13. $\sin\,(x-y)=\sin x\cos y-\cos x\sin y$.

14. $\cos\,(x-y)=\cos x\cos y+\sin x\sin y$.

15. $\tan\,(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$.

16. $\cot\,(x-y)=\dfrac{\cot x\cot y-1}{\cot x+\cot y}$.

17. $\sin\,2x=2\sin x\cos x$.

18. $\cos\,2x=\cos^2 x-\sin^2 x$.

19. $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$.

20. $\cot 2x=\dfrac{\cot^2 x-1}{2\cot x}$.

21. $\sin\dfrac{x}{2}=\pm\sqrt{\dfrac{1-\cos x}{2}}$.

22. $\cos\dfrac{x}{2}=\pm\sqrt{\dfrac{1+\cos x}{2}}$.

23. $\tan\dfrac{x}{2}=\pm\sqrt{\dfrac{1-\cos x}{1+\cos x}}$.

24. $\cot\dfrac{x}{2}=\pm\sqrt{\dfrac{1+\cos x}{1-\cos x}}$.

FORMULAS

The following list of formulas have a wide and general use. The student should know each formula to have a useful knowledge of mathematics.

1. $\pi = 3.1416$.
2. $C = 2\pi r$. C = circumference of a circle, r = radius.
3. $C = \frac{\pi d_1 + d_2}{2}$. C = circumference of ellipse, d_1 and d_2 = long and short diameters.

Areas:

4. Square $A = a^2$. a = side.
5. Rectangle $A = ab$. a and b are the dimensions.
6. Parallelogram $A = bh$. b = base, h = height.
7. Trapezoid $A = \frac{1}{2}h(b+b')$. h = height. b and b' are the parallel sides.
8. Triangle $A = \frac{1}{2}bh$. b = base, h = height.
9. Triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$. a, b and c are the sides, $s = \frac{1}{2}(a+b+c)$.
10. Hexagon $A = 2.598s^2$. s = side.
11. Circle $A = \pi r^2$. r = radius.
12. Ellipse $A = \pi \frac{d_1 d_2}{4}$. d_1 and d_2 = long and short diameters.
13. Surface of sphere $A = 4\pi r_2$. r = radius.

Volumes:

14. Cube $V = a^3$. a = edge.
15. Rectangular solid $V = lwh$, l, w, and h are the dimensions.

16. Rectangular solid, Parallelopiped, Prism or Cylinder. $V=bh$, $b=$area of the base, $h=$height.
17. Cone or Pyramid $V=\frac{1}{3}bh$, $b=$ area of base, $h=$height.
18. Frustum of a Cone or Pyramid.
$V=\frac{1}{3}h(b+b'+\sqrt{bb'})$, b and b' are the areas of the bases and $h=$height.
19. Sphere $V=\frac{4}{3}\pi r^3$. $r=$radius.
20. Spherical segment $V=\frac{1}{2}\pi a\left(r^2+\frac{a^2}{3}\right)$.
$a=$altitude. $r=$radius of the base.

Right Triangle:

21. $c^2=a^2+b^2$. $c=$hypotenuse, a and b the sides.

Quadratic Equation:

22. $$x=\frac{-b\pm\sqrt{b^2+4ac}}{2a}.$$

Trigonometry:

23. Law of sines: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.
24. Law of cosines $a^2=b^2+c^2-2bc\cos A$.

TABLE 1.—DECIMAL EQUIVALENTS

Of Eighths, Sixteenths, Thirty-seconds and Sixty-fourths of an Inch

$\frac{1}{64}$	.015625	$\frac{33}{64}$	.515625
$\frac{1}{32}$	.03125	$\frac{17}{32}$	.53125
$\frac{3}{64}$	.046875	$\frac{35}{64}$	.546875
$\frac{1}{16}$	.0625	$\frac{9}{16}$	.5625
$\frac{5}{64}$	.078125	$\frac{37}{64}$	.578125
$\frac{3}{32}$	.09375	$\frac{19}{32}$	.59375
$\frac{7}{64}$	.109375	$\frac{39}{64}$	.609375
$\frac{1}{8}$	.1250	$\frac{5}{8}$	.6250
$\frac{9}{64}$	.140625	$\frac{41}{64}$	.640625
$\frac{5}{32}$	.15625	$\frac{21}{32}$	.65625
$\frac{11}{64}$	.171875	$\frac{43}{64}$	.671875
$\frac{3}{16}$	.1875	$\frac{11}{16}$	.6875
$\frac{13}{64}$	.203125	$\frac{45}{64}$	.703125
$\frac{7}{32}$	.21875	$\frac{23}{32}$	.71875
$\frac{15}{64}$	.234375	$\frac{47}{64}$	.734375
$\frac{1}{4}$	.2500	$\frac{3}{4}$	.7500
$\frac{17}{64}$	.265625	$\frac{49}{64}$	.765625
$\frac{9}{32}$	.28125	$\frac{25}{32}$	.78125
$\frac{19}{64}$	.296875	$\frac{51}{64}$	.796875
$\frac{5}{16}$	.3125	$\frac{13}{16}$	.8125
$\frac{21}{64}$	.328125	$\frac{53}{64}$	.828125
$\frac{11}{32}$	.34375	$\frac{27}{32}$	.84375
$\frac{23}{64}$	.359375	$\frac{55}{64}$	.859375
$\frac{3}{8}$	.3750	$\frac{7}{8}$	.8750
$\frac{25}{64}$	.390625	$\frac{57}{64}$	.890625
$\frac{13}{32}$	.40625	$\frac{29}{32}$	.90625
$\frac{27}{64}$	.421875	$\frac{59}{64}$	.921875
$\frac{7}{16}$	.4375	$\frac{15}{16}$	.9375
$\frac{29}{64}$	.453125	$\frac{61}{64}$	.953125
$\frac{15}{32}$	.46875	$\frac{31}{32}$	.96875
$\frac{31}{64}$	.484375	$\frac{63}{64}$	.984375
$\frac{1}{2}$	.5000	1	1.0000

n	0	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957
	0	1	2	3	4	5	6	7	8	9

of Numbers from 000 to 999

n	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957
	0	1	2	3	4	5	6	7	8	9

LOG TANGENT

Angle	0′	10′	20′	30′	40′	50′	60′	
0°	−∞	$\bar{3}.46373$	$\bar{3}.76476$	$\bar{3}.94086$	$\bar{2}.06581$	$\bar{2}.16273$	$\bar{2}.24192$	89
1	$\bar{2}.24192$	$\bar{2}.30888$	$\bar{2}.36689$	$\bar{2}.41807$	$\bar{2}.46385$	$\bar{2}.50527$	$\bar{2}.54308$	88
2	$\bar{2}.54308$	$\bar{2}.57788$	$\bar{2}.61009$	$\bar{2}.64009$	$\bar{2}.66816$	$\bar{2}.69453$	$\bar{2}.71940$	87
3	$\bar{2}.71940$	$\bar{2}.74292$	$\bar{2}.76525$	$\bar{2}.78649$	$\bar{2}.80674$	$\bar{2}.82610$	$\bar{2}.84464$	86
4	$\bar{2}.84464$	$\bar{2}.86243$	$\bar{2}.87953$	$\bar{2}.89598$	$\bar{2}.91185$	$\bar{2}.92716$	$\bar{2}.94195$	85°
5°	$\bar{2}.94195$	$\bar{2}.95627$	$\bar{2}.97012$	$\bar{2}.98358$	$\bar{2}.99662$	$\bar{1}.00930$	$\bar{1}.02162$	84
6	$\bar{1}.02162$	$\bar{1}.03361$	$\bar{1}.04528$	$\bar{1}.05666$	$\bar{1}.06775$	$\bar{1}.07858$	$\bar{1}.08914$	83
7	$\bar{1}.08914$	$\bar{1}.09947$	$\bar{1}.10956$	$\bar{1}.11943$	$\bar{1}.12909$	$\bar{1}.13854$	$\bar{1}.14780$	82
8	$\bar{1}.14780$	$\bar{1}.15688$	$\bar{1}.16577$	$\bar{1}.17450$	$\bar{1}.18306$	$\bar{1}.19146$	$\bar{1}.19971$	81
9	$\bar{1}.19971$	$\bar{1}.20782$	$\bar{1}.21578$	$\bar{1}.22361$	$\bar{1}.23130$	$\bar{1}.23887$	$\bar{1}.24632$	80°
10°	$\bar{1}.24632$	$\bar{1}.25365$	$\bar{1}.26086$	$\bar{1}.26797$	$\bar{1}.27496$	$\bar{1}.28186$	$\bar{1}.28865$	79
11	$\bar{1}.28865$	$\bar{1}.29535$	$\bar{1}.30195$	$\bar{1}.30846$	$\bar{1}.31489$	$\bar{1}.32122$	$\bar{1}.32747$	78
12	$\bar{1}.32747$	$\bar{1}.33365$	$\bar{1}.33974$	$\bar{1}.34576$	$\bar{1}.35170$	$\bar{1}.35757$	$\bar{1}.36336$	77
13	$\bar{1}.36336$	$\bar{1}.36909$	$\bar{1}.37476$	$\bar{1}.38035$	$\bar{1}.38589$	$\bar{1}.39136$	$\bar{1}.39677$	76
14	$\bar{1}.39677$	$\bar{1}.40212$	$\bar{1}.40742$	$\bar{1}.41266$	$\bar{1}.41784$	$\bar{1}.42297$	$\bar{1}.42805$	75°
15°	$\bar{1}.42805$	$\bar{1}.43308$	$\bar{1}.43806$	$\bar{1}.44299$	$\bar{1}.44787$	$\bar{1}.45271$	$\bar{1}.45750$	74
16	$\bar{1}.45750$	$\bar{1}.46224$	$\bar{1}.46694$	$\bar{1}.47160$	$\bar{1}.47622$	$\bar{1}.48080$	$\bar{1}.48534$	73
17	$\bar{1}.48534$	$\bar{1}.48984$	$\bar{1}.49430$	$\bar{1}.49872$	$\bar{1}.50311$	$\bar{1}.50746$	$\bar{1}.51178$	72
18	$\bar{1}.51178$	$\bar{1}.51606$	$\bar{1}.52031$	$\bar{1}.52452$	$\bar{1}.52870$	$\bar{1}.53285$	$\bar{1}.53697$	71
19	$\bar{1}.53697$	$\bar{1}.54106$	$\bar{1}.54512$	$\bar{1}.54915$	$\bar{1}.55315$	$\bar{1}.55712$	$\bar{1}.56107$	70°
20°	$\bar{1}.56107$	$\bar{1}.56498$	$\bar{1}.56887$	$\bar{1}.57274$	$\bar{1}.57658$	$\bar{1}.58039$	$\bar{1}.58418$	69
21	$\bar{1}.58418$	$\bar{1}.58794$	$\bar{1}.59168$	$\bar{1}.59540$	$\bar{1}.59909$	$\bar{1}.60276$	$\bar{1}.60641$	68
22	$\bar{1}.60641$	$\bar{1}.61004$	$\bar{1}.61364$	$\bar{1}.61722$	$\bar{1}.62079$	$\bar{1}.62433$	$\bar{1}.62785$	67
23	$\bar{1}.62785$	$\bar{1}.63135$	$\bar{1}.63484$	$\bar{1}.63830$	$\bar{1}.64175$	$\bar{1}.64517$	$\bar{1}.64858$	66
24	$\bar{1}.64858$	$\bar{1}.65197$	$\bar{1}.65535$	$\bar{1}.65870$	$\bar{1}.66204$	$\bar{1}.66537$	$\bar{1}.66867$	65°
25°	$\bar{1}.66867$	$\bar{1}.67196$	$\bar{1}.67524$	$\bar{1}.67850$	$\bar{1}.68174$	$\bar{1}.68497$	$\bar{1}.68818$	64
26	$\bar{1}.68818$	$\bar{1}.69138$	$\bar{1}.69457$	$\bar{1}.69774$	$\bar{1}.70089$	$\bar{1}.70404$	$\bar{1}.70717$	63
27	$\bar{1}.70717$	$\bar{1}.71028$	$\bar{1}.71339$	$\bar{1}.71648$	$\bar{1}.71955$	$\bar{1}.72262$	$\bar{1}.72567$	62
28	$\bar{1}.72567$	$\bar{1}.72872$	$\bar{1}.73175$	$\bar{1}.73476$	$\bar{1}.73777$	$\bar{1}.74077$	$\bar{1}.74375$	61
29	$\bar{1}.74375$	$\bar{1}.74673$	$\bar{1}.74969$	$\bar{1}.75264$	$\bar{1}.75558$	$\bar{1}.75852$	$\bar{1}.76144$	60°
30°	$\bar{1}.76144$	$\bar{1}.76435$	$\bar{1}.76725$	$\bar{1}.77015$	$\bar{1}.77303$	$\bar{1}.77591$	$\bar{1}.77877$	59
31	$\bar{1}.77877$	$\bar{1}.78163$	$\bar{1}.78448$	$\bar{1}.78732$	$\bar{1}.79015$	$\bar{1}.79297$	$\bar{1}.79579$	58
32	$\bar{1}.79579$	$\bar{1}.79860$	$\bar{1}.80140$	$\bar{1}.80419$	$\bar{1}.80697$	$\bar{1}.80975$	$\bar{1}.81252$	57
33	$\bar{1}.81252$	$\bar{1}.81528$	$\bar{1}.81803$	$\bar{1}.82078$	$\bar{1}.82352$	$\bar{1}.82626$	$\bar{1}.82899$	56
34	$\bar{1}.82899$	$\bar{1}.83171$	$\bar{1}.83442$	$\bar{1}.83713$	$\bar{1}.83984$	$\bar{1}.84254$	$\bar{1}.84523$	55°
35°	$\bar{1}.84523$	$\bar{1}.84791$	$\bar{1}.85059$	$\bar{1}.85327$	$\bar{1}.85594$	$\bar{1}.85860$	$\bar{1}.86126$	54
36	$\bar{1}.86126$	$\bar{1}.86392$	$\bar{1}.86656$	$\bar{1}.86921$	$\bar{1}.87185$	$\bar{1}.87448$	$\bar{1}.87711$	53
37	$\bar{1}.87711$	$\bar{1}.87974$	$\bar{1}.88236$	$\bar{1}.88498$	$\bar{1}.88759$	$\bar{1}.89020$	$\bar{1}.89281$	52
38	$\bar{1}.89281$	$\bar{1}.89541$	$\bar{1}.89801$	$\bar{1}.90061$	$\bar{1}.90320$	$\bar{1}.90578$	$\bar{1}.90837$	51
39	$\bar{1}.90837$	$\bar{1}.91095$	$\bar{1}.91353$	$\bar{1}.91610$	$\bar{1}.91868$	$\bar{1}.92125$	$\bar{1}.92381$	50°
40°	$\bar{1}.92381$	$\bar{1}.92638$	$\bar{1}.92894$	$\bar{1}.93150$	$\bar{1}.93406$	$\bar{1}.93661$	$\bar{1}.93916$	49
41	$\bar{1}.93916$	$\bar{1}.94171$	$\bar{1}.94426$	$\bar{1}.94681$	$\bar{1}.94935$	$\bar{1}.95190$	$\bar{1}.95444$	48
42	$\bar{1}.95444$	$\bar{1}.95698$	$\bar{1}.95952$	$\bar{1}.96205$	$\bar{1}.96459$	$\bar{1}.96712$	$\bar{1}.96966$	47
43	$\bar{1}.96966$	$\bar{1}.97219$	$\bar{1}.97472$	$\bar{1}.97725$	$\bar{1}.97978$	$\bar{1}.98231$	$\bar{1}.98484$	46
44	$\bar{1}.98484$	$\bar{1}.98737$	$\bar{1}.98989$	$\bar{1}.99242$	$\bar{1}.99495$	$\bar{1}.99747$	0.00000	45°
	60′	50′	40′	30′	20′	10′	0′	Angle

LOG COTANGENT

of Sines and Cosines

LOG SINE

Angle	0′	10′	20′	30′	40′	50′	60′	
45°	$\bar{1}$.84949	$\bar{1}$.85074	$\bar{1}$.85200	$\bar{1}$.85324	$\bar{1}$.85448	$\bar{1}$.85571	$\bar{1}$.85693	44
46	$\bar{1}$.85693	$\bar{1}$.85815	$\bar{1}$.85936	$\bar{1}$.86056	$\bar{1}$.86176	$\bar{1}$.86295	$\bar{1}$.86413	43
47	$\bar{1}$.86413	$\bar{1}$.86530	$\bar{1}$.86647	$\bar{1}$.86762	$\bar{1}$.86879	$\bar{1}$.86993	$\bar{1}$.87107	42
48	$\bar{1}$.87107	$\bar{1}$.87221	$\bar{1}$.87334	$\bar{1}$.87446	$\bar{1}$.87557	$\bar{1}$.87668	$\bar{1}$.87778	41
49	$\bar{1}$.87778	$\bar{1}$.87887	$\bar{1}$.87996	$\bar{1}$.88105	$\bar{1}$.88212	$\bar{1}$.88319	$\bar{1}$.88425	40°
50°	$\bar{1}$.88425	$\bar{1}$.88531	$\bar{1}$.88636	$\bar{1}$.88741	$\bar{1}$.88844	$\bar{1}$.88948	$\bar{1}$.89050	39
51	$\bar{1}$.89050	$\bar{1}$.89152	$\bar{1}$.89254	$\bar{1}$.89354	$\bar{1}$.89455	$\bar{1}$.89554	$\bar{1}$.89653	38
52	$\bar{1}$.89653	$\bar{1}$.89752	$\bar{1}$.89849	$\bar{1}$.89947	$\bar{1}$.90043	$\bar{1}$.90139	$\bar{1}$.90235	37
53	$\bar{1}$.90235	$\bar{1}$.90330	$\bar{1}$.90424	$\bar{1}$.90518	$\bar{1}$.90611	$\bar{1}$.90704	$\bar{1}$.90796	36
54	$\bar{1}$.90796	$\bar{1}$.90887	$\bar{1}$.90978	$\bar{1}$.91069	$\bar{1}$.91158	$\bar{1}$.91248	$\bar{1}$.91336	35°
55°	$\bar{1}$.91336	$\bar{1}$.91425	$\bar{1}$.91512	$\bar{1}$.91599	$\bar{1}$.91686	$\bar{1}$.91772	$\bar{1}$.91857	34
56	$\bar{1}$.91857	$\bar{1}$.91942	$\bar{1}$.92027	$\bar{1}$.92111	$\bar{1}$.92194	$\bar{1}$.92277	$\bar{1}$.92359	33
57	$\bar{1}$.92359	$\bar{1}$.92441	$\bar{1}$.92522	$\bar{1}$.92603	$\bar{1}$.92683	$\bar{1}$.92763	$\bar{1}$.92842	32
58	$\bar{1}$.92842	$\bar{1}$.92921	$\bar{1}$.92999	$\bar{1}$.93077	$\bar{1}$.93154	$\bar{1}$.93230	$\bar{1}$.93307	31
59	$\bar{1}$.93307	$\bar{1}$.93382	$\bar{1}$.93457	$\bar{1}$.93532	$\bar{1}$.93606	$\bar{1}$.93680	$\bar{1}$.93753	30°
60°	$\bar{1}$.93753	$\bar{1}$.93826	$\bar{1}$.93898	$\bar{1}$.93970	$\bar{1}$.94041	$\bar{1}$.94112	$\bar{1}$.94182	29
61	$\bar{1}$.94182	$\bar{1}$.94252	$\bar{1}$.94321	$\bar{1}$.94390	$\bar{1}$.94458	$\bar{1}$.94526	$\bar{1}$.94593	28
62	$\bar{1}$.94593	$\bar{1}$.94660	$\bar{1}$.94727	$\bar{1}$.94793	$\bar{1}$.94858	$\bar{1}$.94923	$\bar{1}$.94988	27
63	$\bar{1}$.94988	$\bar{1}$.95052	$\bar{1}$.95116	$\bar{1}$.95179	$\bar{1}$.95242	$\bar{1}$.95304	$\bar{1}$.95366	26
64	$\bar{1}$.95366	$\bar{1}$.95427	$\bar{1}$.95488	$\bar{1}$.95549	$\bar{1}$.95609	$\bar{1}$.95668	$\bar{1}$.95728	25°
65°	$\bar{1}$.95728	$\bar{1}$.95786	$\bar{1}$.95844	$\bar{1}$.95902	$\bar{1}$.95960	$\bar{1}$.96017	$\bar{1}$.96073	24
66	$\bar{1}$.96073	$\bar{1}$.96129	$\bar{1}$.96185	$\bar{1}$.96240	$\bar{1}$.96294	$\bar{1}$.96349	$\bar{1}$.96403	23
67	$\bar{1}$.96403	$\bar{1}$.96456	$\bar{1}$.96509	$\bar{1}$.96562	$\bar{1}$.96614	$\bar{1}$.96665	$\bar{1}$.96717	22
68	$\bar{1}$.96717	$\bar{1}$.96767	$\bar{1}$.96818	$\bar{1}$.96868	$\bar{1}$.96917	$\bar{1}$.96966	$\bar{1}$.97015	21
69	$\bar{1}$.97015	$\bar{1}$.97063	$\bar{1}$.97111	$\bar{1}$.97159	$\bar{1}$.97206	$\bar{1}$.97252	$\bar{1}$.97299	20°
70°	$\bar{1}$.97299	$\bar{1}$.97344	$\bar{1}$.97390	$\bar{1}$.97435	$\bar{1}$.97479	$\bar{1}$.97523	$\bar{1}$.97567	19
71	$\bar{1}$.97567	$\bar{1}$.97610	$\bar{1}$.97653	$\bar{1}$.97696	$\bar{1}$.97738	$\bar{1}$.97779	$\bar{1}$.97821	18
72	$\bar{1}$.97821	$\bar{1}$.97861	$\bar{1}$.97902	$\bar{1}$.97942	$\bar{1}$.97982	$\bar{1}$.98021	$\bar{1}$.98060	17
73	$\bar{1}$.98060	$\bar{1}$.98098	$\bar{1}$.98136	$\bar{1}$.98174	$\bar{1}$.98211	$\bar{1}$.98248	$\bar{1}$.98284	16
74	$\bar{1}$.98284	$\bar{1}$.98320	$\bar{1}$.98356	$\bar{1}$.98391	$\bar{1}$.98426	$\bar{1}$.98460	$\bar{1}$.98494	15°
75°	$\bar{1}$.98494	$\bar{1}$.98528	$\bar{1}$.98561	$\bar{1}$.98594	$\bar{1}$.98627	$\bar{1}$.98659	$\bar{1}$.98690	14
76	$\bar{1}$.98690	$\bar{1}$.98722	$\bar{1}$.98753	$\bar{1}$.98783	$\bar{1}$.98813	$\bar{1}$.98843	$\bar{1}$.98872	13
77	$\bar{1}$.98872	$\bar{1}$.98901	$\bar{1}$.98930	$\bar{1}$.98958	$\bar{1}$.98986	$\bar{1}$.99013	$\bar{1}$.99040	12
78	$\bar{1}$.99040	$\bar{1}$.99067	$\bar{1}$.99093	$\bar{1}$.99119	$\bar{1}$.99145	$\bar{1}$.99170	$\bar{1}$.99195	11
79	$\bar{1}$.99195	$\bar{1}$.99219	$\bar{1}$.99243	$\bar{1}$.99267	$\bar{1}$.99290	$\bar{1}$.99313	$\bar{1}$.99335	10°
80°	$\bar{1}$.99335	$\bar{1}$.99357	$\bar{1}$.99379	$\bar{1}$.99400	$\bar{1}$.99421	$\bar{1}$.99442	$\bar{1}$.99462	9
81	$\bar{1}$.99462	$\bar{1}$.99482	$\bar{1}$.99501	$\bar{1}$.99520	$\bar{1}$.99539	$\bar{1}$.99557	$\bar{1}$.99575	8
82	$\bar{1}$.99575	$\bar{1}$.99593	$\bar{1}$.99610	$\bar{1}$.99627	$\bar{1}$.99643	$\bar{1}$.99659	$\bar{1}$.99675	7
83	$\bar{1}$.99675	$\bar{1}$.99690	$\bar{1}$.99705	$\bar{1}$.99720	$\bar{1}$.99734	$\bar{1}$.99748	$\bar{1}$.99761	6
84	$\bar{1}$.99761	$\bar{1}$.99775	$\bar{1}$.99787	$\bar{1}$.99800	$\bar{1}$.99812	$\bar{1}$.99823	$\bar{1}$.99834	5°
85°	$\bar{1}$.99834	$\bar{1}$.99845	$\bar{1}$.99856	$\bar{1}$.99866	$\bar{1}$.99876	$\bar{1}$.99885	$\bar{1}$.99894	4
86	$\bar{1}$.99894	$\bar{1}$.99903	$\bar{1}$.99911	$\bar{1}$.99919	$\bar{1}$.99926	$\bar{1}$.99934	$\bar{1}$.99940	3
87	$\bar{1}$.99940	$\bar{1}$.99947	$\bar{1}$.99953	$\bar{1}$.99959	$\bar{1}$.99964	$\bar{1}$.99969	$\bar{1}$.99974	2
88	$\bar{1}$.99974	$\bar{1}$.99978	$\bar{1}$.99982	$\bar{1}$.99985	$\bar{1}$.99988	$\bar{1}$.99991	$\bar{1}$.99993	1
89	$\bar{1}$.99993	$\bar{1}$.99995	$\bar{1}$.99997	$\bar{1}$.99998	$\bar{1}$.99999	0.00000	0.00000	0°
	60′	50′	40′	30′	20′	10′	0′	Angle

LOG COSINE

LOG SINE

Angle	0′	10′	20′	30′	40′	50′	60′	
0°	. — ∞	$\bar{3}$.46373	$\bar{3}$.76475	$\bar{3}$.94084	$\bar{2}$.06578	$\bar{2}$.16268	$\bar{2}$.24186	89
1	$\bar{2}$.24186	$\bar{2}$.30879	$\bar{2}$.36678	$\bar{2}$.41792	$\bar{2}$.46366	$\bar{2}$.50504	$\bar{2}$.54282	88
2	$\bar{2}$.54282	$\bar{2}$.57757	$\bar{2}$.60973	$\bar{2}$.63963	$\bar{2}$.66769	$\bar{2}$.69400	$\bar{2}$.71880	87
3	$\bar{2}$.71880	$\bar{2}$.74226	$\bar{2}$.76451	$\bar{2}$.78568	$\bar{2}$.80585	$\bar{2}$.82513	$\bar{2}$.84358	86
4	$\bar{2}$.84358	$\bar{2}$.86128	$\bar{2}$.87829	$\bar{2}$.89464	$\bar{2}$.91040	$\bar{2}$.92561	$\bar{2}$.94030	85°
5°	$\bar{2}$.94030	$\bar{2}$.95450	$\bar{2}$.96825	$\bar{2}$.98157	$\bar{2}$.99450	$\bar{1}$.00704	$\bar{1}$.01923	84
6	$\bar{1}$.01923	$\bar{1}$.03109	$\bar{1}$.04262	$\bar{1}$.05386	$\bar{1}$.06481	$\bar{1}$.07548	$\bar{1}$.08589	83
7	$\bar{1}$.08589	$\bar{1}$.09606	$\bar{1}$.10599	$\bar{1}$.11570	$\bar{1}$.12519	$\bar{1}$.13447	$\bar{1}$.14356	82
8	$\bar{1}$.14356	$\bar{1}$.15245	$\bar{1}$.16116	$\bar{1}$.16970	$\bar{1}$.17807	$\bar{1}$.18628	$\bar{1}$.19433	81
9	$\bar{1}$.19433	$\bar{1}$.20223	$\bar{1}$.20999	$\bar{1}$.21761	$\bar{1}$.22509	$\bar{1}$.23244	$\bar{1}$.23967	80°
10°	$\bar{1}$.23967	$\bar{1}$.24677	$\bar{1}$.25376	$\bar{1}$.26063	$\bar{1}$.26739	$\bar{1}$.27405	$\bar{1}$.28060	79
11	$\bar{1}$.28060	$\bar{1}$.28705	$\bar{1}$.29340	$\bar{1}$.29966	$\bar{1}$.30582	$\bar{1}$.31189	$\bar{1}$.31788	78
12	$\bar{1}$.31788	$\bar{1}$.32378	$\bar{1}$.32960	$\bar{1}$.33534	$\bar{1}$.34100	$\bar{1}$.34658	$\bar{1}$.35209	77
13	$\bar{1}$.35209	$\bar{1}$.35752	$\bar{1}$.36289	$\bar{1}$.36819	$\bar{1}$.37341	$\bar{1}$.37858	$\bar{1}$.38368	76
14	$\bar{1}$.38368	$\bar{1}$.38871	$\bar{1}$.39369	$\bar{1}$.39860	$\bar{1}$.40346	$\bar{1}$.40825	$\bar{1}$.41300	75°
15°	$\bar{1}$.41300	$\bar{1}$.41768	$\bar{1}$.42232	$\bar{1}$.42690	$\bar{1}$.43143	$\bar{1}$.43591	$\bar{1}$.44034	74
16	$\bar{1}$.44034	$\bar{1}$.44472	$\bar{1}$.44905	$\bar{1}$.45334	$\bar{1}$.45758	$\bar{1}$.46178	$\bar{1}$.46594	73
17	$\bar{1}$.46594	$\bar{1}$.47005	$\bar{1}$.47411	$\bar{1}$.47814	$\bar{1}$.48213	$\bar{1}$.48607	$\bar{1}$.48998	72
18	$\bar{1}$.48998	$\bar{1}$.49385	$\bar{1}$.49768	$\bar{1}$.50148	$\bar{1}$.50523	$\bar{1}$.50896	$\bar{1}$.51264	71
19	$\bar{1}$.51264	$\bar{1}$.51629	$\bar{1}$.51991	$\bar{1}$.52350	$\bar{1}$.52705	$\bar{1}$.53056	$\bar{1}$.53405	70°
20°	$\bar{1}$.53405	$\bar{1}$.53751	$\bar{1}$.54093	$\bar{1}$.54433	$\bar{1}$.54769	$\bar{1}$.55102	$\bar{1}$.55433	69
21	$\bar{1}$.55433	$\bar{1}$.55761	$\bar{1}$.56085	$\bar{1}$.56408	$\bar{1}$.56727	$\bar{1}$.57044	$\bar{1}$.57358	68
22	$\bar{1}$.57358	$\bar{1}$.57669	$\bar{1}$.57978	$\bar{1}$.58284	$\bar{1}$.58588	$\bar{1}$.58889	$\bar{1}$.59188	67
23	$\bar{1}$.59188	$\bar{1}$.59484	$\bar{1}$.59778	$\bar{1}$.60070	$\bar{1}$.60359	$\bar{1}$.60646	$\bar{1}$.60931	66
24	$\bar{1}$.60931	$\bar{1}$.61214	$\bar{1}$.61494	$\bar{1}$.61773	$\bar{1}$.62049	$\bar{1}$.62323	$\bar{1}$.62595	65°
25°	$\bar{1}$.62595	$\bar{1}$.62865	$\bar{1}$.63133	$\bar{1}$.63398	$\bar{1}$.63662	$\bar{1}$.63924	$\bar{1}$.64184	64
26	$\bar{1}$.64184	$\bar{1}$.64442	$\bar{1}$.64698	$\bar{1}$.64953	$\bar{1}$.65205	$\bar{1}$.65456	$\bar{1}$.65705	63
27	$\bar{1}$.65705	$\bar{1}$.65952	$\bar{1}$.66197	$\bar{1}$.66441	$\bar{1}$.66682	$\bar{1}$.66922	$\bar{1}$.67161	62
28	$\bar{1}$.67161	$\bar{1}$.67398	$\bar{1}$.67633	$\bar{1}$.67866	$\bar{1}$.68098	$\bar{1}$.68328	$\bar{1}$.68557	61
29	$\bar{1}$.68557	$\bar{1}$.68784	$\bar{1}$.69010	$\bar{1}$.69234	$\bar{1}$.69456	$\bar{1}$.69677	$\bar{1}$.69897	60°
30°	$\bar{1}$.69897	$\bar{1}$.70115	$\bar{1}$.70332	$\bar{1}$.70547	$\bar{1}$.70761	$\bar{1}$.70973	$\bar{1}$.71184	59
31	$\bar{1}$.71184	$\bar{1}$.71393	$\bar{1}$.71602	$\bar{1}$.71809	$\bar{1}$.72014	$\bar{1}$.72218	$\bar{1}$.72421	58
32	$\bar{1}$.72421	$\bar{1}$.72622	$\bar{1}$.72823	$\bar{1}$.73022	$\bar{1}$.73219	$\bar{1}$.73416	$\bar{1}$.73611	57
33	$\bar{1}$.73611	$\bar{1}$.73805	$\bar{1}$.73997	$\bar{1}$.74189	$\bar{1}$.74379	$\bar{1}$.74568	$\bar{1}$.74756	56
34	$\bar{1}$.74756	$\bar{1}$.74943	$\bar{1}$.75128	$\bar{1}$.75313	$\bar{1}$.75496	$\bar{1}$.75678	$\bar{1}$.75859	55°
35°	$\bar{1}$.75859	$\bar{1}$.76039	$\bar{1}$.76218	$\bar{1}$.76395	$\bar{1}$.76572	$\bar{1}$.76747	$\bar{1}$.76922	54
36	$\bar{1}$.76922	$\bar{1}$.77095	$\bar{1}$.77268	$\bar{1}$.77439	$\bar{1}$.77609	$\bar{1}$.77778	$\bar{1}$.77946	53
37	$\bar{1}$.77946	$\bar{1}$.78113	$\bar{1}$.78280	$\bar{1}$.78445	$\bar{1}$.78609	$\bar{1}$.78772	$\bar{1}$.78934	52
38	$\bar{1}$.78934	$\bar{1}$.79095	$\bar{1}$.79256	$\bar{1}$.79415	$\bar{1}$.79573	$\bar{1}$.79731	$\bar{1}$.79887	51
39	$\bar{1}$.79887	$\bar{1}$.80043	$\bar{1}$.80197	$\bar{1}$.80351	$\bar{1}$.80504	$\bar{1}$.80656	$\bar{1}$.80807	50°
40°	$\bar{1}$.80807	$\bar{1}$.80957	$\bar{1}$.81106	$\bar{1}$.81254	$\bar{1}$.81402	$\bar{1}$.81549	$\bar{1}$.81694	49
41	$\bar{1}$.81694	$\bar{1}$.81839	$\bar{1}$.81983	$\bar{1}$.82126	$\bar{1}$.82269	$\bar{1}$.82410	$\bar{1}$.82551	48
42	$\bar{1}$.82551	$\bar{1}$.82691	$\bar{1}$.82830	$\bar{1}$.82968	$\bar{1}$.83106	$\bar{1}$.83242	$\bar{1}$.83378	47
43	$\bar{1}$.83378	$\bar{1}$.83513	$\bar{1}$.83648	$\bar{1}$.83781	$\bar{1}$ 83914	$\bar{1}$.84046	$\bar{1}$.84177	46
44	$\bar{1}$.84177	$\bar{1}$.84308	$\bar{1}$.84437	$\bar{1}$.84566	$\bar{1}$.84694	$\bar{1}$.84822	$\bar{1}$.84949	45°
	60′	50′	40′	30′	20′	10′	0′	Angle

LOG COSINE

of Tangents and Cotangents

LOG TANGENT

Angle	0′	10′	20′	30′	40′	50′	60′	
45°	0.00000	0.00253	0.00505	0.00758	0.01011	0.01263	0.01516	44
46	0.01516	0.01769	0.02022	0.02275	0.02528	0.02781	0.03034	43
47	0.03034	0.03288	0.03541	0.03795	0.04048	0.04302	0.04556	42
48	0.04556	0.04810	0.05065	0.05319	0.05574	0.05829	0.06084	41
49	0.06084	0.06339	0.06594	0.06850	0.07106	0.07362	0.07619	40°
50°	0.07619	0.07875	0.08132	0.08390	0.08647	0.08905	0.09163	39
51	0.09163	0.09422	0.09680	0.09939	0.10199	0.10459	0.10719	38
52	0.10719	0.10980	0.11241	0.11502	0.11764	0.12026	0.12289	37
53	0.12289	0.12552	0.12815	0.13079	0.13344	0.13608	0.13874	36
54	0.13874	0.14140	0.14406	0.14673	0.14941	0.15209	0.15477	35°
55°	0.15477	0.15746	0.16016	0.16287	0.16558	0.16829	0.17101	34
56	0.17101	0.17374	0.17648	0.17922	0.18197	0.18472	0.18748	33
57	0.18748	0.19025	0.19303	0.19581	0.19860	0.20140	0.20421	32
58	0.20421	0.20703	2.20985	0.21268	0.21552	0.21837	0.22123	31
59	0.22123	0.22409	0.22697	0.22985	0.23275	0.23565	0.23856	30°
60°	0.23856	0.24148	0.24442	0.24736	0.25031	0.25327	0.25625	29
61	0.25625	0.25923	0.26223	0.26524	0.26825	0.27128	0.27433	28
62	0.27433	0.27738	0.28045	0.28352	0.28661	0.28972	0.29283	27
63	0.29283	0.29596	0.29911	0.30226	0.30543	0.30862	0.31182	26
64	0.31182	0.31503	0.31826	0.32150	0.32476	0.32804	0.33133	25°
65°	0.33133	0.33463	0.33796	0.34130	0.34465	0.34803	0.35142	24
66	0.35142	0.35483	0.35825	0.36170	0.36516	0.36865	0.37215	23
67	0.37215	0.37567	0.37921	0.38278	0.38636	0.38996	0.39359	22
68	0.39359	0.39724	0.40091	0.40460	0.40832	0.41206	0.41582	21
69	0.41582	0.41961	0.42342	0.42726	0.43113	0.43502	0.43893	20°
70°	0.43893	0.44288	0.44685	0.45085	0.45488	0.45894	0.46303	19
71	0.46303	0.46715	0.47130	0.47548	0.47969	0.48394	0.48822	18
72	0.48822	0.49254	0.49689	0.50128	0.50570	0.51016	0.51466	17
73	0.51466	0.51920	0.52378	0.52840	0.53306	0.53776	0.54250	16
74	0.54250	0.54729	0.55213	0.55701	0.56194	0.56692	0.57195	15°
75°	0.57195	0.57703	0.58216	0.58734	0.59258	0.59788	0.60323	14
76	0.60323	0.60864	0.61411	0.61965	0.62524	0.63091	0.63664	13
77	0.63664	0.64243	0.64830	0.65424	0.66026	0.66635	0.67253	12
78	0.67253	0.67878	0.68511	0.69154	0.69805	0.70465	0.71135	11
79	0.71135	0.71814	0.72504	0.73203	0.73914	0.74635	0.75368	10°
80°	0.75368	0.76113	0.76870	0.77639	0.78422	0.79218	0.80029	9
81	0.80029	0.80854	0.81694	0.82550	0.83423	0.84312	0.85220	8
82	0.85220	0.86146	0.87091	0.88057	0.89044	0.90053	0.91086	7
83	0.91086	0.92142	0.93225	0.94334	0.95472	0.96639	0.97838	6
84	0.97838	0.99070	1.00338	1.01642	1.02987	1.04373	1.05805	5°
85°	1.05805	1.07284	1.08815	1.10402	1.12047	1.13757	1.15536	4
86	1.15536	1.17390	1.19326	1.21351	1.23475	1.25708	1.28060	3
87	1.28060	1.30547	1.33184	1.35991	1.38991	1.42212	1.45692	2
88	1.45692	1.49473	1.53615	1.58193	1.63311	1.69112	1.75808	1
89	1.75808	1.83727	1.93419	2.05914	2.23524	2.53627	∞	0°
	60′	50′	40′	30′	20′	10′	0′	Angle

LOG COTANGENT

Natural Sines

SINE

Angle	0′	10′	20′	30′	40′	50′	60′	
0°	0.00000	0.00291	0.00582	0.00873	0.01164	0.01454	0.01745	89
1	0.01745	0.02036	0.02327	0.02618	0.02908	0.03199	0.03490	88
2	0.03490	0.03781	0.04071	0.04362	0.04653	0.04943	0.05234	87
3	0.05234	0.05524	0.05814	0.06105	0.06395	0.06685	0.06976	86
4	0.06976	0.07266	0.07556	0.07847	0.08136	0.08426	0.08716	85°
5°	0.08716	0.09005	0.09295	0.09585	0.09874	0.10164	0.10453	84
6	0.10453	0.10742	0.11031	0.11320	0.11609	0.11898	0.12187	83
7	0.12187	0.12476	0.12764	0.13053	0.13341	0.13629	0.13917	82
8	0.13917	0.14205	0.14493	0.14781	0.15069	0.15356	0.15643	81
9	0.15643	0.15931	0.16218	0.16505	0.16792	0.17078	0.17365	80°
10°	0.17365	0.17651	0.17937	0.18224	0.18509	0.18795	0.19081	79
11	0.19081	0.19366	0.19652	0.19937	0.20222	0.20507	0.20791	78
12	0.20791	0.21076	0.21360	0.21644	0.21928	0.22212	0.22495	77
13	0.22495	0.22778	0.23062	0.23345	0.23627	0.23910	0.24192	76
14	0.24192	0.24474	0.24756	0.25038	0.25320	0.25601	0.25882	75°
15°	0.25882	0.26163	0.26443	0.26724	0.27004	0.27284	0.27564	74
16	0.27564	0.27843	0.28123	0.28402	0.28680	0.28959	0.29237	73
17	0.29237	0.29515	0.29793	0.30071	0.30348	0.30625	0.30902	72
18	0.30902	0.31178	0.31454	0.31730	0.32006	0.32282	0.32557	71
19	0.32557	0.32832	0.33106	0.33381	0.33655	0.33929	0.34202	70°
20°	0.34202	0.34475	0.34748	0.35021	0.35293	0.35565	0.35837	69
21	0.35837	0.36108	0.36379	0.36650	0.36921	0.37191	0.37461	68
22	0.37461	0.37730	0.37999	0.38268	0.38537	0.38805	0.39073	67
23	0.39073	0.39341	0.39608	0.39875	0.40142	0.40408	0.40674	66
24	0.40674	0.40939	0.41204	0.41469	0.41734	0.41998	0.42262	65°
25°	0.42262	0.42525	0.42788	0.43051	0.43313	0.43575	0.43837	64
26	0.43837	0.44098	0.44359	0.44620	0.44880	0.45140	0.45399	63
27	0.45399	0.45658	0.45917	0.46175	0.46433	0.46690	0.46947	62
28	0.46947	0.47204	0.47460	0.47716	0.47971	0.48226	0.48481	61
29	0.48481	0.48735	0.48989	0.49242	0.49495	0.49748	0.50000	60°
30°	0.50000	0.50252	0.50503	0.50754	0.51004	0.51254	0.51504	59
31	0.51504	0.51753	0.52002	0.52250	0.52498	0.52745	0.52992	58
32	0.52992	0.53238	0.53484	0.53730	0.53975	0.54220	0.54464	57
33	0.54464	0.54708	0.54951	0.55194	0.55436	0.55678	0.55919	56
34	0.55919	0.56160	0.56401	0.56641	0.56880	0.57119	0.57358	55°
35°	0.57358	0.57596	0.57833	0.58070	0.58307	0.58543	0.58779	54
36	0.58779	0.59014	0.59248	0.59482	0.59716	0.59949	0.60182	53
37	0.60182	0.60414	0.60645	0.60876	0.61107	0.61337	0.61566	52
38	0.61566	0.61795	0.62024	0.62251	0.62479	0.62706	0.62932	51
39	0.62932	0.63158	0.63383	0.63608	0.63832	0.64056	0.64279	50°
40°	0.64279	0.64501	0.64723	0.64945	0.65166	0.65386	0.65606	49
41	0.65606	0.65825	0.66044	0.66262	0.66480	0.66697	0.66913	48
42	0.66913	0.67129	0.67344	0.67559	0.67773	0.67987	0.68200	47
43	0.68200	0.68412	0.68624	0.68835	0.69046	0.69256	0.69466	46
44	0.69466	0.69675	0.69883	0.70091	0.70298	0.70505	0.70711	45
	60′	50′	40′	30′	20′	10′	0′	Angle

COSINE

and Cosines

SINE

Angle	0′	10′	20′	30′	40′	50′	60′	
45°	0.70711	0.70916	0.71121	0.71325	0.71529	0.71732	0.71934	44
46	0.71934	0.72136	0.72337	0.72537	0.72737	0.72937	0.73135	43
47	0.73135	0.73333	0.73531	0.73728	0.73924	0.74120	0.74314	42
48	0.74314	0.74509	0.74703	0.74896	0.75088	0.75280	0.75471	41
49	0.75471	0.75661	0.75851	0.76041	0.76229	0.76417	0.76604	40°
50°	0.76604	0.76791	0.76977	0.77162	0.77347	0.77531	0.77715	39
51	0.77715	0.77897	0.78079	0.78261	0.78442	0.78622	0.78801	38
52	0.78801	0.78980	0.79158	0.79335	0.79512	0.79688	0.79864	37
53	0.79864	0.80038	0.80212	0.80386	0.80558	0.80730	0.80902	36
54	0.80902	0.81072	0.81242	0.81412	0.81580	0.81748	0.81915	35°
55°	0.81915	0.82082	0.82248	0.82413	0.82577	0.82741	0.82904	34
56	0.82904	0.83066	0.83228	0.83389	0.83549	0.83708	0.83867	33
57	0.83867	0.84025	0.84182	0.84339	0.84495	0.84650	0.84805	32
58	0.84805	0.84959	0.85112	0.85264	0.85416	0.85567	0.85717	31
59	0.85717	0.85866	0.86015	0.86163	0.86310	0.86457	0.86603	30°
60°	0.86603	0.86748	0.86892	0.87036	0.87178	0.87321	0.87462	29
61	0.87462	0.87603	0.87743	0.87882	0.88020	0.88158	0.88295	28
62	0.88295	0.88431	0.88566	0.88701	0.88835	0.88968	0.89101	27
63	0.89101	0.89232	0.89363	0.89493	0.89623	0.89752	0.89879	26
64	0.89879	0.90007	0.90133	0.90259	0.90383	0.90507	0.90631	25°
65°	0.90631	0.90753	0.90875	0.90996	0.91116	0.91236	0.91355	24
66	0.91355	0.91472	0.91590	0.91706	0.91822	0.91936	0.92050	23
67	0.92050	0.92164	0.92276	0.92388	0.92499	0.92609	0.92718	22
68	0.92718	0.92827	0.92935	0.93042	0.93148	0.93253	0.93358	21
69	0.93358	0.93462	0.93565	0.93667	0.93769	0.93869	0.93969	20°
70°	0.93969	0.94068	0.94167	0.94264	0.94361	0.94457	0.94552	19
71	0.94552	0.94646	0.94740	0.94832	0.94924	0.95015	0.95106	18
72	0.95106	0.95195	0.95284	0.95372	0.95459	0.95545	0.95630	17
73	0.95630	0.95715	0.95799	0.95882	0.95964	0.96046	0.96126	16
74	0.96126	0.96206	0.96285	0.96363	0.96440	0.96517	0.96593	15°
75°	0.96593	0.96667	0.96742	0.96815	0.96887	0.96959	0.97030	14
76	0.97030	0.97100	0.97169	0.97237	0.97304	0.97371	0.97437	13
77	0.97437	0.97502	0.97566	0.97630	0.97692	0.97754	0.97815	12
78	0.97815	0.97875	0.97934	0.97992	0.98050	0.98107	0.98163	11
79	0.98163	0.98218	0.98272	0.98325	0.98378	0.98430	0.98481	10°
80°	0.98481	0.98531	0.98580	0.98629	0.98676	0.98723	0.98769	9
81	0.98769	0.98814	0.98858	0.98902	0.98944	0.98986	0.99027	8
82	0.99027	0.99067	0.99106	0.99144	0.99182	0.99219	0.99255	7
83	0.99255	0.99290	0.99324	0.99357	0.99390	0.99421	0.99452	6
84	0.99452	0.99482	0.99511	0.99540	0.99567	0.99594	0.99619	5°
85°	0.99619	0.99644	0.99668	0.99692	0.99714	0.99736	0.99756	4
86	0.99756	0.99776	0.99795	0.99813	0.99831	0.99847	0.99863	3
87	0.99863	0.99878	0.99892	0.99905	0.99917	0.99929	0.99939	2
88	0.99939	0.99949	0.99958	0.99966	0.99973	0.99979	0.99985	1
89	0.99985	0.99989	0.99993	0.99996	0.99998	1.00000	1.00000	0°
	60′	50′	40′	30′	20′	10′	0′	Angle

COSINE

Natural Tangents

TANGENT

Angle	0′	10′	20′	30′	40′	50′	60′	
0°	0.00000	0.00291	0.00582	0.00873	0.01164	0.01455	0.01746	89
1	0.01746	0.02036	0.02328	0.02619	0.02910	0.03201	0.03492	88
2	0.03492	0.03783	0.04075	0.04366	0.04658	0.04949	0.05241	87
3	0.05241	0.05533	0.05824	0.06116	0.06408	0.06700	0.06993	86
4	0.06993	0.07285	0.07578	0.07870	0.08163	0.08456	0.08749	85°
5°	0.08749	0.09042	0.09335	0.09629	0.09923	0.10216	0.10510	84
6	0.10510	0.10805	0.11099	0.11394	0.11688	0.11983	0.12278	83
7	0.12278	0.12574	0.12869	0.13165	0.13461	0.13758	0.14054	82
8	0.14054	0.14351	0.14648	0.14945	0.15243	0.15540	0.15838	81
9	0.15838	0.16137	0.16435	0.16734	0.17033	0.17333	0.17633	80°
10°	0.17633	0.17933	0.18233	0.18534	0.18835	0.19136	0.19438	79
11	0.19438	0.19740	0.20042	0.20345	0.20648	0.20952	0.21256	78
12	0.21256	0.21560	0.21864	0.22169	0.22475	0.22781	0.23087	77
13	0.23087	0.23393	0.23700	0.24008	0.24316	0.24624	0.24933	76
14	0.24933	0.25242	0.25552	0.25862	0.26172	0.26483	0.26795	75°
15°	0.26795	0.27107	0.27419	0.27732	0.28046	0.28360	0.28675	74
16	0.28675	0.28990	0.29305	0.29621	0.29938	0.30255	0.30573	73
17	0.30573	0.30891	0.31210	0.31530	0.31850	0.32171	0.32492	72
18	0.32492	0.32814	0.33136	0.33460	0.33783	0.34108	0.34433	71
19	0.34433	0.34758	0.35085	0.35412	0.35740	0.36068	0.36397	70°
20°	0.36397	0.36727	0.37057	0.37388	0.37720	0.38053	0.38386	69
21	0.38386	0.38721	0.39055	0.39391	0.39727	0.40065	0.40403	68
22	0.40403	0.40741	0.41081	0.41421	0.41763	0.42105	0.42447	67
23	0.42447	0.42791	0.43136	0.43481	0.43828	0.44175	0.44523	66
24	0.44523	0.44872	0.45222	0.45573	0.45924	0.46277	0.46631	65°
25°	0.46631	0.46985	0.47341	0.47698	0.48055	0.48414	0.48773	64
26	0.48773	0.49134	0.49495	0.49858	0.50222	0.50587	0.50953	63
27	0.50953	0.51320	0.51688	0.52057	0.52427	0.52798	0.53171	62
28	0.53171	0.53545	0.53920	0.54296	0.54673	0.55051	0.55431	61
29	0.55431	0.55812	0.56194	0.56577	0.56962	0.57348	0.57735	60°
30°	0.57735	0.58124	0.58513	0.58905	0.59297	0.59691	0.60086	59
31	0.60086	0.60483	0.60881	0.61280	0.61681	0.62083	0.62487	58
32	0.62487	0.62892	0.63299	0.63707	0.64117	0.64528	0.64941	57
33	0.64941	0.65355	0.65771	0.66189	0.66608	0.67028	0.67451	56
34	0.67451	0.67875	0.68301	0.68728	0.69157	0.69588	0.70021	55°
35°	0.70021	0.70455	0.70891	0.71329	0.71769	0.72211	0.72654	54
36	0.72654	0.73100	0.73547	0.73996	0.74447	0.74900	0.75355	53
37	0.75355	0.75812	0.76272	0.76733	0.77196	0.77661	0.78129	52
38	0.78129	0.78598	0.79070	0.79544	0.80020	0.80498	0.80978	51
39	0.80978	0.81461	0.81946	0.82434	0.82923	0.83415	0.83910	50°
40°	0.83910	0.84407	0.84906	0.85408	0.85912	0.86419	0.86929	49
41	0.86929	0.87441	0.87955	0.88473	0.88992	0.89515	0.90040	48
42	0.90040	0.90569	0.91099	0.91633	0.92170	0.92709	0.93252	47
43	0.93252	0.93797	0.94345	0.94896	0.95451	0.96008	0.96569	46
44	0.96569	0.97133	0.97700	0.98270	0.98843	0.99420	1.00000	45°
	60′	50′	40′	30′	20′	10′	0′	Angle

COTANGENT

and Cotangents

TANGENT

Angle	0′	10′	20′	30′	40′	50′	60′	
45°	1.00000	1.00583	1.01170	1.01761	1.02355	1.02952	1.03553	44
46	1.03553	1.04158	1.04766	1.05378	1.05994	1.06613	1.07237	43
47	1.07237	1.07864	1.08496	1.09131	1.09770	1.10414	1.11061	42
48	1.11061	1.11713	1.12369	1.13029	1.13694	1.14363	1.15037	41
49	1.15037	1.15715	1.16398	1.17085	1.17777	1.18474	1.19175	40°
50°	1.19175	1.19882	1.20593	1.21310	1.22031	1.22758	1.23490	39
51	1.23490	1.24227	1.24969	1.25717	1.26471	1.27230	1.27994	38
52	1.27994	1.28764	1.29541	1.30323	1.31110	1.31904	1.32704	37
53	1.32704	1.33511	1.34323	1.35142	1.35968	1.36800	1.37638	36
54	1.37638	1.38484	1.39336	1.40195	1.41061	1.41934	1.42815	35°
55°	1.42815	1.43703	1.44598	1.45501	1.46411	1.47330	1.48256	34
56	1.48256	1.49190	1.50133	1.51084	1.52043	1.53010	1.53987	33
57	1.53987	1.54972	1.55966	1.56969	1.57981	1.59002	1.60033	32
58	1.60033	1.61074	1.62125	1.63185	1.64256	1.65337	1.66428	31
59	1.66428	1.67530	1.68643	1.69766	1.70901	1.72047	1.73205	30°
60°	1.73205	1.74375	1.75556	1.76749	1.77955	1.79174	1.80405	29
61	1.80405	1.81649	1.82906	1.84177	1.85462	1.86760	1.88073	28
62	1.88073	1.89400	1.90741	1.92098	1.93470	1.94858	1.96261	27
63	1.96261	1.97680	1.99116	2.00569	2.02039	2.03526	2.05030	26
64	2.05030	2.06553	2.08094	2.09654	2.11233	2.12832	2.14451	25°
65°	2.14451	2.16090	2.17749	2.19430	2.21132	2.22857	2.24604	24
66	2.24604	2.26374	2.28167	2.29984	2.31826	2.33693	2.35585	23
67	2.35585	2.37504	2.39449	2.41421	2.43422	2.45451	2.47509	22
68	2.47509	2.49597	2.51715	2.53865	2.56046	2.58261	2.60509	21
69	2.60509	2.62791	2.65109	2.67462	2.69853	2.72281	2.74748	20°
70°	2.74748	2.77254	2.79802	2.82391	2.85023	2.87700	2.90421	19
71	2.90421	2.93189	2.96004	2.98869	3.01783	3.04749	3.07768	18
72	3.07768	3.10842	3.13972	3.17159	3.20406	3.23714	3.27085	17
73	3.27085	3.30521	3.34023	3.37594	3.41236	3.44951	3.48741	16
74	3.48741	3.52609	3.56557	3.60588	3.64705	3.68909	3.73205	15°
75°	3.73205	3.77595	3.82083	3.86671	3.91364	3.96165	4.01078	14
76	4.01078	4.06107	4.11256	4.16530	4.21933	4.27471	4.33148	13
77	4.33148	4.38969	4.44942	4.51071	4.57363	4.63825	4.70463	12
78	4.70463	4.77286	4.84300	4.91516	4.98940	5.06584	5.14455	11
79	5.14455	5.22566	5.30928	5.39552	5.48451	5.57638	5.67128	10°
80°	5.67128	5.76937	5.87080	5.97576	6.08444	6.19703	6.31375	9
81	6.31375	6.43484	6.56055	6.69116	6.82694	6.96823	7.11537	8
82	7.11537	7.26873	7.42871	7.59575	7.77035	7.95302	8.14435	7
83	8.14435	8.34496	8.55555	8.77689	9.00983	9.25530	9.51436	6
84	9.51436	9.78817	10.0780	10.3854	10.7119	11.0594	11.4301	5°
85°	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	4
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1
89	57.2900	68.7501	85.9398	114.589	171.885	343.774	∞	0°
	60′	50′	40′	30′	20′	10′	0′	Angle

COTANGENT

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